# *Mathematics and Rhetoric*

## *Jacques Peletier, Guillaume Gosselin*

## *and*

## *The Making of the French Algebraic Tradition*

by

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***Abstract***

In sixteenth century Paris a circle of mathematicians produced texts of the most advanced kind of algebra. This "French algebraic tradition" will be the context for Viète's symbolic algebra. Comparing French algebraic texts with Italian and German, and examining the publishing context (mathematical and otherwise), I establish a periodization in two phases.

Jacques Peletier stands for the introduction of the abacus tradition (elementary commercial arithmetic) and algebra at the court. Peletier's algebraic program is connected to his theory of rhetoric. He establishes a genre of texts devoted to algebra in vernacular, promoting French as a scientific language. Rhetorical criteria order ***L'Algèbre***, emphasizing structure and theory (i.e. definitions and demonstrations). He innovates on Cardano's and Stifel's treatments of equations in several unknowns.

Phase two changes style and content; Guillaume Gosselin is representative. Manuals shift focus from problems and questions to equations and classification. Solution of equations becomes the purpose of algebra. Problems are no longer addressed as such, but conceived in their most general form as corresponding equations. Algebrists connect with a milieu of jurists important in politics and scholarship. This erudite milieu favored the recovery of Diophantus' ***Arithmetic***, which provides a set of theoretical problems on numbers. Diophantus solved problems by transforming them into particular cases; Gosselin reaches a general solution by transforming them into equations. Gosselin's notation makes the difference. The algebrists create an illustrious genealogy for algebra, deriving it from Greece (Diophantus) rather than abacus schools.

Three features of this tradition (late abacus algebra, rhetoric, and genealogy) can be traced back to Italian humanism: Cardano's and Tartaglia's algebra, *imitatio* (translation to a new vernacular learned culture), and the construction of a history for the discipline. The French algebrists radically transformed all three. Their *translatio* authorized them to abandon links to the medieval tradition and to build a new discipline that they could see as national. Preparing the adoption of this discipline by the legal élite was the rhetorical interpretation of logic developed in Paris at the time. This provided a theoretical frame in which generalized algebraic problems were seen as Cicero's *quaestiones infinitae*, i.e. scientific questions.

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## Introduction

### *1. The "algebraic revolution"*

One of the certainties of the history of algebra is that this branch of mathematics experienced a period of great development between the end of the sixteenth and the beginning of the seventeenth century. This development allowed western algebra to constitute itself as a discipline and to become *ars analytica*, to use Viète's expression, or, in modern terminology, symbolic algebra and the theory of equations. As a consequence, this development had a major impact on the writing not only of geometry, but all the mathematical sciences. Mathematical manuals as well as new mathematical results were now written in algebraic language. By the end of the seventeenth century the new disciplines of analytic geometry, calculus, and rational mechanics had arisen, and they can be considered a development of the use of algebra on a large scale, as a tool for the investigation of various mathematical problems. The same could be said of number theory, which already at the time of Fermat's method, around 1628, was a field of application for algebra. We may say, more generally, that Fermat's number theory depended on his knowledge of algebra and the related program of algebraization of problems.[1] In fact, the consequences of this transformation of algebra spread far and wide into European science and culture. We will be therefore justified in speaking of the "algebraic revolution."

Symbolic algebra is usually understood to have appeared in two remarkable episodes: Viète's symbolic algebra and analytic program, and Descartes' theory of equations and scheme of human knowledge based on mathematics.

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1 See Mahoney 1973 and Goldstein 1990.

Both authors are extolled as radical and heroic innovators. It has been widely accepted that they acted against, rather than within, a tradition. This view has been sustained by the fact that these authors did not acknowledge their predecessors or affiliations. This is a problem to which I shall devote much attention in Chapter 6 below. Here we need only suggest that the prevalence of this view has left the mathematical background of both theories, and the reliance of both authors on their context, largely unexplored. The neglect of this topic is typically justified by the claim that algebra before the radical innovations of Viète and Descartes can explain nothing of their discoveries. Stated in this summary way, we can easily see that this assertion cannot be sustained, for it can only be proven after contextual research has been thoroughly pursued. Thus, this claim cannot justify from the outset our not taking seriously the French mathematical milieu of the sixteenth century. Moreover, all algebraic developments in this period are usually grouped together under the broad heading of "cossist algebra," which includes roughly anything that happened before Viète. The origin of this term, which will be important for us as the argument advances, lies with German authors of the sixteenth century. For them, "cossist algebra" indicated calculations with the unknown "cossa" developed in a style taken (as the term *cosa* or "thing" suggests) from the Italians.

It is not our intention to deny the importance of Viète and Descartes. The transformation they wrought was so great that it made algebra unrecognizable. This was at least one of the reasons why Viète's work was not understood until a few decades after its appearance in 1591. And it is precisely the importance of their achievement that leads us to ask how it was accomplished. Whatever the answer, one thing is certain: they innovated *within* a tradition, not outside of it.

My research has shown that before Viète and Descartes, great changes were already taking place *within* the complex and manifold algebraic tradition of the six$^{\text{teenth}}$ century. This was particularly true in France, a fact the importance of which must not be underestimated. Let us take just one example. The only source admitted for Viète was Diophantus, the Alexandrian mathematician. But why was *France* the only context in which Diophantus' ***Arithmetic*** was actually transformed into symbolic algebra? Many circumstances make this unique conjuncture unlikely. Diophantus had already been rediscovered in the fifteenth century; his ***Arithmetic*** had been put to work by Bombelli in Italy and Steven in the Netherlands. Yet, it was the French who made the switch.

I shall argue that it was the particular milieu of mathematics in France, especially a half-century-long tradition of work on algebra, together with cultural and political projects and means for its dissemination, that provided the necessary context for first Viète and then Descartes. Thus, the central purpose of this work is to investigate some aspects of the French algebraic tradition before Viète. It must first of all be proven that such a tradition existed as such, with later authors drawing on earlier ones. In addition, I shall provide a first periodization for it. Finally, I shall suggest that the French algebraic tradition did not merely serve as the background for later, greater, inventions. Rather, the authors, texts, and practices within that tradition contributed directly some of those features which we recognize as central to the new symbolic algebra typically assigned to Viète and Descartes.

So far, we have not mentioned the main source for algebra, Arabic mathematics. Therefore, as a preface to the main part of our story, it will be helpful to take a step backwards. Before the sixteenth century, a long and distinct tradition involving the use of and teaching about the abacus was the main vehicle for the transmission of algebra from the

Arabic world into the West. This "abacus tradition," as we shall refer to it here, provided one of the cornerstones of the French algebraic tradition which is our main concern.

## *2. The role of the abacus tradition*

Historians generally acknowledge the direct dependence of algebra on the commercial tradition, in particular on the genre of commercial arithmetic. In this respect, we are presented with an historiographical thesis analogous to the one concerning the process by which other *arts* became disciplines, such as dissection or ballistics. The general thesis is that the remarkable transformation which took place in this period is preceded by a burst of technical development by artisans, and by a corresponding improvement in their status. In the case of algebra, the relevant version of this thesis is as follows: the first discoveries which determined the remarkable subsequent developments in western algebra, such as the formulas for the equations of third and fourth degrees, were made in the abacus schools, which is to say outside the realm of universities.

What do we know about this alternative educational institution, the abacus school? From the start, it should be kept in mind that the abacus *school* was the main locus for and propagator of the abacus *tradition* which concerns us here.

Thanks to recent studies,[2] we know more now than a few years ago. In Italy, Spain, and the German countries this kind of school was founded rather early, around the thirteenth century. They were organized by merchants and their goal was training for business. Schooling, which had both written and oral components, was conducted almost entirely in the vernacular. The abacus school represented for its students an alternative to the

2See in particular the work of Franci and Toti Rigatelli, by W. Van Egmond, T. Lévy, and some recent studies on Chuquet (see Hay 1988.

university, and played a key part in the formation of an "intermediate cultural stratum"[3] between the illiterate and the Latinists. We know also that the social group which transmitted and developed algebra before it entered the university curriculum, was constituted by the arithmetic teachers in the abacus schools. The schools had already begun to transmit at least some algebra already at the time of direct Arabic influence. In the abacus texts, algebra appeared at most in a chapter, or in a repertoire of problems which it could be used solve. In this context, western algebra developed for centuries on the basis of the original transmission from East to West, i.e. Fibonacci's ***Liber abaci***,[4] and Abu Kamil[5].

Abacus schools were the place where Arabic algebra was preserved and developed. By contrast, the universities taught only Euclid's ***Elements***, together with simple arithmetic of both sorts, *speculativa* and *practica*. In fact, recent research has even established that at the universities the main source, besides geometry, was Sacrobosco's *algorismus*, which included the four operations for natural and rational numbers according to Arabic numerals.[6]

The later acceleration in Europe of results in algebra leading to the solution of third degree equations and then to symbolic algebra took place in parallel with analogous developments in algebra occurring in the Arabic countries up to the fifteenth century.[7] Of

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3 See for the introduction and the definition of this phrase, Carlo Maccagni. ***Lo 'strato culturale intermedio' e il Rinascimento***. In "La Filosofia della scienza oggi", Istituto Italiano per gli studi filosofici, aprile 1991.

4 Written in 1202. This text shows Fibonacci's knowledge of Al Kwarizmi, Al Karaji and Abu Kamil.

5 See the text, and the discussion on the diffusion of this work from the Latin school of Toledo in Abu Kamil, 1935.

6 See G. R. Evans, 1977.

7 See in this respect the general works of Roshdi Rashed, such as ***Entre Arithmétique et Algèbre. Recherches sur l'histoire des mathématiques arabes***. Paris, Les Belles Lettres, 1984

great interest are comparisons between the early research in symbolism in the Western Arabic world, culminating with Al Qalasadi's use of letters in calculation. However, no trace of direct transmission has been found.

While this is not the place to discuss the social context of the western part of the Arabic world, or the possible channels of transmission,[8] we can state some facts about abacus schools in Italy and the German countries. In particular, around the end of the fifteenth century we notice an important change in the abacus schools. As printing developed, the audience for the abacus text was transformed. Three genres emerged: the treatise for students, the manual for teachers, and the handbook for artisans. In this period, we see for the first time a sort of "permeability" between the abacus schools and the universities.

This "period of permeability" has many aspects. We have already mentioned that a few practical arts[9] entered the university curriculum, and in general the status of artisans of all specialties improved. Without reporting on the vast literature on this topic, we shall simply recall that many types of technicians and their techniques acquired a new social role, not only in the Italian Quattrocento, but also at courts and in cities across Europe. Furthermore, new careers were opened to artisans besides the actual practice of their craft. They began, for example, to publish manuals describing techniques for other artisans to follow. A similar shift opened new careers, outside the universities, for intellectuals who mastered an art. They began to publish manuals describing artiginal techniques to a learned public. A reflection of this transformation can be found in many books published in the

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8 On this, see Tony Lévy's research on the presence of Hebrew manuscripts in Italy.

9 See for instance Paolo Rossi. ***I filosofi e le macchine. 1400-1700***. Milano, Feltrinelli, 1962.

mid-sixteenth century in France. These include numerous texts in French, and therefore not acceptable at the university, which nonetheless ended up being read by students. Thus, they became part of the important learned corpus despite the university establishment. Ambroise Paré and Bernard Palissy are among the most influential of the artisans turned author, while some of the algebrists central to the present study, like Peletier, are typical of the learned popularizers. A strong analogy to algebra is music, another field in which "popular" culture entered the written and learned tradition in sixteenth-century France.

But let us return to the case of algebra and its transmission, and in particular to two figures representative of this general "period of permeability" between "knowledge contexts" -- Nicolas Chuquet and Luca Pacioli.

The case of Chuquet is particularly interesting when we remember that very little is known about arithmetic and algebra in France before the sixteenth century. The requisite manuscripts are simply not available. Recent studies[10] have determined the existence of two areas of abacus schools in fifteenth-century France, one around Paris and Normandie and the other one around Lyon and the Occitan area. (The texts are in Provençal.)

As for Chuquet personally, we now know that he was a master of abacus school in Lyon, but that before writing his ***Triparty***[11] he had also studied in Paris, where the abacus schools were more closely connected to university teaching.[12] In this work Chuquet collects

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10 See in particular Guy Beaujouan. ***The place of Nicolas Chuquet in the typology of fifteenth century French arithmetics***. In ***Mathematics from manuscript to print, 1300-1600***. éd. Cynthia Hay, Oxford, Clarendon Press, 1988. pp. 73-88.

11 ***Triparty en la science des nombres***. The *Bibliothéque Nationale* contains a manuscript of this work from 1484.

12 See Paul Benoit. ***The commercial arithmetic of Nicolas Chuquet***. In ***Mathematics from manuscript to print***. pp.96-116.

the abacus school knowledge of arithmetic and algebra, and includes his own results. It has been shown[13] that the problems he treats had a practical use at the time.

Pacioli is another typical representative of the period of permeability. He was a humanist and professor at the university, but he was so interested in the abacus tradition that he studied and elaborated the manuscript of the ***Liber abaci*** by Fibonacci. Later, he published a text containing a large quantity of abacus mathematics, the ***Summa de arithmetica, geometria, proportioni et proportionalita*** (Venice 1494).

There are other examples, both in Italy and in other countries. In Italy, we can recall Scipione del Ferro, to whom we ascribe the sixteenth-century version of the solution of third degree equations. He was professor at the university of Bologna, but his pupil Antonio Maria Fiore was a "maestro d'abaco." Nicolò Tartaglia was another typical technician who taught himself Latin and wrote exclusively in Italian. He taught at an abacus school but, when publishing his ***General Trattato***[14] in the genre of the texts for abacus schools, did so in a way that was also suitable for university students. On the other hand, he was in contact with Cardano, who was a professor of medicine and philosophy (but not algebra) at the university. But this was later in the century. The point is that already by 1530 the "permeability" of knowledge contexts for both persons and texts is remarkable. So, while we cannot *a priori* suppose the teaching, or even any knowledge, of algebra on the part of a university professor, we do in fact find remarkable instances of it. This is particularly true in France, a country which did not have a strong abacus tradition. By contrast, in the German

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13 Paul Benoit. "The commercial arithmetic of Nicolas Chuquet." Dans ***Mathematics from manuscript to print***. cité, pp.96-116.

14 Nicolò Tartaglia. ***General trattato di numeri e misure***. Venise, C. Troiano dei Navò, 1560.

countries or in Italy some knowledge of algebra can be supposed. A very important consequence follows from this. The absence of a widespread tradition of abacus schools in France means that the abacus texts are imported and even translated directly into a university context, as we shall see in the case of the importation of Cardano, or Gosselin's translation of Tartaglia.

Furthermore, by 1560 France had a flourishing Collège Royal, where humanist culture together with humanist points of view on popular and national culture were debated and supported. Our working hypothesis is that these conditions are relevant to, and even decisive for, the development of the French algebraic tradition that reached a hegemonic position with Viète and Descartes.

Clearly, the abacus schools were for centuries the context of transmission and development of algebra. However, the sixteenth-century results in algebra that we consider especially important are not only the fruit of this milieu. Rather, they arise during this period from the permeability between the abacus schools and the universities, in Italy and the German countries. One could say that this period of permeability explains the new theoretical tendency of the manuals, which we already find in Pacioli, and certainly in Cardano. It is in this context that Cardano wrote the first work entirely devoted to algebra in 1545.[15] Here Algebra is presented in Latin for the first time.

Secondly, some features of algebra depend not so much on the solution formulas, but on the different conceptual structure introduced at this time in algebraic texts. This appears to be the result of a further process, i.e. the introduction of algebra into France, after its late transformation in the other countries.

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15 Girolamo Cardano. ***Ars magna***. Nuremberg, J. Petreius, 1545.

This working hypothesis corrects the view stated earlier, according to which Viète gained his competence in algebra from books in the cossic tradition, directly derived from the *abacus'* tradition, on one hand, and from Diophantus on the other. While it is true that he cites only Cardano and Diophantus, we shall see here that these two authors were not simply available to him without a context, but were transmitted and interpreted through the French algebraic tradition. About Descartes, we know that his skills were learned in part from the manual of Clavius, and we attribute the rest to the time he spent in Holland and Germany. Thus, we suppose that Viète's and Descartes' intellectual formation and activity took place independently from the process by which their subject was adopted by the universities in Italy and Germany and the intellectual élite in Paris, which is to say without a filter which would modify the presentation of the subject and make evident new theoretical, as well as cultural, aims. My thesis is that, precisely in the case of algebra, that process characteristic of the "period of permeability," i.e. the constitution of a discipline from an art, should be viewed as a fundamental part of the transformation, the second step being the completion of this process in France. French algebrists arrived at symbolic algebra through innovations in the style of texts. This change does not occur only in terms of the passage from "syncopated algebra" to symbolic algebra. I mean this, in addition, in the sense that the texts cease to be repertoires of problems and their solutions, and become instead treatises on the theory of equations. Finally, I mean this also in the sense that the influence of symbolic algebra on the seventeenth century did not consist only in the diffusion of technical innovations, but more precisely in a new way to present mathematical results.

### *3. What is symbolic algebra?*

In fact, this way of rethinking the history of algebra entails that we give a definition to symbolic algebra as the point of arrival. In identifying it, as we usually do, with the works of Viète and Descartes, we will have as points of reference the following aspects:

1) the introduction of a symbolism that allows the treatment of general equations, which is to say different letters for the unknowns on the one hand and the coefficients (or known terms) on the other

2) the determination of solution formulas for equations of the third and fourth degree

And then further, with respect to the theory of equations:

3) the elaboration of techniques for the reduction of equations to some standard cases

4) the determination of relations between coefficients and roots

5) the determination of number of roots, the theorem of factorization, and the method of indeterminate coefficients.

If the whole of this represents symbolic algebra, and can serve as a definition, each detail of the picture shows, instead, a certain "instability", in the sense that these points cannot be taken as describing a "stable state" of the discipline, even after Descartes. These were, rather, the consequences of a gradual development from the sixteenth to the late seventeenth century.

Concerning the first point, where there is a calculus of polynomials all algebra has an operative symbolism. Viète introduced symbolism for the coefficients and the known terms in his first algebraic work, ***In artem analyticen Isagoge***.[16] However, we must recall that he

16 Tours, J. Mettayer, 1591.

did not fully exploit this tool, he did not make a radical reduction in the case of the equations he considers, or at least not as radical as we could expect after the introduction of his symbolism. To put this another way, his equations are not really general equations in our sense. A simple example which to our eyes seems useless is the distinction between

*A quad. + B in A aequetur S plano*

and

*A quad. - B in A aequetur S plano*

Besides, it is not necessary to believe that "cossic algebra" had no way to treat general equations. As Diophantus had already done, it was enough to fix an arbitrary numerical value and adhere to it, and in that way reduce and resolve the equation. Cardano had already arrived at a limited number of cases of equations by employing a general verbal description (for example: "the cube plus the thing is equal to number") and treating particular examples. We will see that the Parisian algebrists attain a higher degree of generality although always by making use of a verbal description. More generally, we will see that these algebrists made many innovations in the area of symbolism which were not without consequence.

The most typical achievement of the French algebrists is the choice of letters for several unknowns. They transformed the tradition in this way, by emphasizing the importance of the technique, far more than previous authors had done.

Turning to the second point, the solution of equations of the third and fourth degree is attributed (at least in the modern West) to two Italian mathematicians, Scipione del Ferro and Lodovico Ferrari. It was transmitted to France through the work of Cardano. Thus, this was not a new result for Viète and Descartes.

The third point is the reduction to standard equations: in this domain, Viète developed to a great extent techniques that one finds, for example, in Cardano and Bombelli.

The fourth point is the expression of coefficients in terms of roots; an example here is the fact that the second coefficient is equal to the sum of the roots with the sign changed. This part of the theory of equations was amply developed by Viète, but one finds traces of it in Cardano and Peletier develops this aspect in his text.

Finally, we come to the point concerning Cartesian factorization, which was probably Descartes' principal contribution.

I hope that this schema can at least suggest two points, first, that symbolic algebra is a useful model, but that it took form little by little, and secondly, that this process appears much more clearly if we look precisely at the changes in **form**. In other words, the changes which open a space for symbolic algebra were changes within a particular genre and had a more significant impact on the structure of the theory or the simple manipulation of symbols than on immediate solutions. Properly understood, this development of the theory also gave rise to improvements regarding the possibilities for solutions.

### *4. The existence of a French tradition*

All I have said so far contributes to a picture in which Viète would have been preceded by a relevant French tradition in algebra, and would have been inspired by it. Another question is whether he did actually receive the algebra of his French contemporaries.

It is believed that François Viète, the accepted founder of symbolic algebra, made contacts with all the most prominent mathematicians when he went to Paris for the first time, in the years 1571-1573 and then in his second stay, from 1580 to 1584. This view is offered by Ritter in his famous article of 1895 in the ***Revue occidentale philosophique, sociale et politique***, and repeated by Hofmann in his 1970 introduction to Viète's works.[17] In particular, Ritter and Hofman confidently assert that Viète met with Ramus, Forcadel, Peletier, Errard, Foix-Candalle, Gosselin, Monantheuil. This information is of course of particular relevance if we want to uncover the formative setting of some of the problems that Viète later resolved. Unfortunately, neither Ritter nor Hofmann give the source of their certainty about Viète's contacts. Lacking such evidence, we must reconstruct both the connections within the Parisian group of mathematicians and their connections with Viète.

In this work, I will give the first description of the French algebraic tradition before Viète and of its periodization. In order to do so, I shall focus on two main authors more specifically involved in the publication of algebraic texts, Jacques Peletier and Guillaume Gosselin. The first is the founder of the French algebraic tradition, while the second represents the moment of institutionalization.

***5. Methodology***

Thus far I have discussed the main purpose of this study: to show that in sixteenth-century France all the algebraic sources relevant for the famous founders of symbolic algebra had already been absorbed and reelaborated. Now I wish to make explicit some points of method.

First of all, this study belongs to the history of mathematics, for its main purpose is

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17 See Vieta.***Opera***.

to give an account of the development of a mathematical theory, i.e. symbolic algebra. However, an adequate account requires that the context of production and diffusion of this theory be explored. Thus, the books and their uses will be given a very special place. This will allow us also to establish a main line of social and material connection among the various people involved in the creation of the French algebraic tradition.

Furthermore, we have already indicated that the main changes of the process taking place in France are changes in the *form* of the art of algebra. Thus, we shall stress developments in the change of perspective *on* the discipline as much as, if not at times more than, developments *within* the discipline itself. The means by which we may track this self-understanding of the discipline is the development of rhetorical theory taking place in France during this period. This is not only crucial in itself, but is connected to many aspects of sixteenth-century French culture, of which the most relevant here are taken to be theory of logic, philosophy of mathematics, historical theory and over all the re-elaboration of Cicero's philosophy.

Furthermore, the rhetorical aspect of the story evidences the nexus between the disciplinary and the social aspects of the argument. For, in sixteenth-century French algebra the use of rhetoric is not limited to the presentation of the discipline. Nor is it just a philosophy of mathematics (the theorization of a *mathesis universalis*) designed to promote the status of the art in the institutions of high culture. Rhetoric is more than a persuasive coating or an argumentative superstructure cobbled onto a complete edifice in the "context of justification." It becomes, rather, an essential element in the formulation of techniques and disciplinary practices. Thus, it will not suffice to focus only on questions of "genre" or "style", even though these categories will be part of our way of looking at the *corpus* of algebra texts. Our aim will be to recognize in mathematical authors their own categories of *inventio*, *dispositio* and *elocutio* in action. Our purpose is to bring to light the practical role played by rhetorical strategy for authors who explicitly and implicitly believed that rhetoric was crucial to their enterprise. More generally, rhetoric served the purpose of producing books suited to a certain intellectual milieu where algebra had not yet appeared as a part of mathematics. But certainly rhetoric intervened at the level of discovery by providing a legitimation for a certain direction for thought. The most important point, however, is that in this way rhetoric also set the conditions for mathematical problems, and so determined their form.

There are other reasons to give a central place in this study to the rhetorical aspect. As a strategy for thought rhetoric stood as a filter for technical and symbolic forms. Yet, if rhetoric was the culturally legitimate way of thinking, not just for a few but for all the people involved in the algebraic world, it seems likely that various types of social exchange took place in this medium. In fact, rhetoric was a cultural program that shaped choices in writing, university politics, and personal careers, as we see in the examples of Peletier and Ramus, though the same was already true for Erasmus. This is the wider sense in which the term will be taken. It is in this sense that the rhetorical aspect of the development of French algebra is limited by the disciplinary aspect on one side and by the social aspect on the other, and in turn shapes them both.

Furthermore, the association of algebra and rhetoric, or rather their identification, was a line of thought present in the French algebraic tradition, insofar as it could found the program of generalized use of algebra in the sciences. For, the dialectical parts of ancient rhetoric could replace Aristotelian logic. Thus, algebraic texts can be seen in connection with contemporary theories of method.

Recent historiography of science has stressed two aspects of the scientific revolution: experimental science and mechanicism. Transformations in mathematical thought and in the notions and practices of mathematization have been taken as less important. For instance, the nexus between juridical rhetoric and mathematics in the seventeenth century has been seen in terms of proof, and not in terms of problems and solutions.

On the other hand, many recent studies in the history of science have stressed the constructive and rhetorical aspect of strategies of persuasion in the sciences. This approach contributed to clarify the "external" development of scientific theories. I suggest, in this work, seeing rhetoric in the sixteenth-century sense, as a context for discourse on science but also as a form of scientific discourse, a strategy for formulating problems and researching their solutions. Rhetoric can bring to light the institutional and social relations which have created the conditions for the sciences. Furthermore, the rhetoric of the formulation of problems gives form, already at the stage of discovery, to the theoretical content, before any problem of legitimation arises. The rhetoric of the formulation of problems is, in this sense, a means of communication between the theoretical and the socio-cultural aspects of science.

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# Chapter 1

## The algebrists and their context

***Part A: Jacques Peletier du Mans and his humanist project for algebra***

Jacques Peletier du Mans is well-known as an important figure in the culture of sixteenth-century France, but he is known primarily for his literary work -- his poetic activity, translations of the classics, and promotion of a new orthographic system. The importance of his scientific work, in geometry, arithmetic, and algebra is less well-known. Yet, it is his scientific contributions that make him one of the major French figures of the century, in particular his role in the formation of an algebraic tradition, of which he was its conscious founder. He contributed to it by publishing two works, ***L'Arithmétique***, which was a significant transformation of commercial arithmetics of the time, with some relevance to algebra, and an algebraic treatise, ***L'Algèbre***. Both the French and Latin editions of this work received wide diffusion.

A chronology of the facts of his life and works was compiled Jugé,[18] together with a bibliography. Both have been revised by further biographical studies, in particular those by N.Z. Davis[19] and J.J. Verdonk.[20] More recent works have focussed on some specific area of Peletier's interests. Of special relevance here are the aspects of Peletier's life, works and social connections that relate to his scientific activity, and in particular, to his humanist project for algebra. These will be discussed in connection with the three centers of the French intellectual life of the period: the *Court*, the *publishers*, and the *colleges*.

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18 See C. P. Jugé, 1907.

19 I shall mention other works by Davis giving information about Peletier, but the essay that illuminates many biographical points is "Peletier and Beza part company", ***Studies in the Renaissance***, XI (1964).

20 See the entry for Peletier, Jacques in ***The Dictionary of Scientific Biography***

***1. Peletier's early career: The Court and the Academies***

The French court of the sixteenth century is not a usual topic for the history of science, and the relation between the French court and science in our period has not been explored nearly as much as that of the following century. Yet, it is acknowledged that the court was a context shared, one way or another, by sixteenth-century scientists, the very authors who instructed the first scientific generation of the seventeenth century. In addition, there are some scientific achievements specific to the sixteenth century that developed within groups associated with the court. This is the case for symbolic algebra. Here, we shall attempt to show how Peletier's projects were related to his connection to the court.

One commom way of organizing gatherings on scientific topics at the court was through the academies. But historians are not certain about the extent to which the academies pursued scientific topics. In 1947, Frances Yates wrote:

> The possible relationship between late sixteenth- and early seventeenth-century musical humanism, and early seventeenth-century science has not, -- I believe, -- been investigated."[21]

Yates at the same time stresses the importance of a historiography devoted to the development of musical theory at the sixteenth-century French court and the lack of attention to the connection between this historical fact and the development of other sciences. In fact, Yates has shown that the French Academies of the sixteenth century should be considered the true ancestors of seventeenth-century academies, for here the whole range of the mathematical sciences was represented; the Académie de Baïf and the Académie du Palais are good examples. These academies focussed on music, of course, but

21 See Yates 1947. Since 1947, of course, historiography has made some progress. In particular, we should remember the book by Sealy, 1981, on the Palace Academy.

in many ways this is true of other mathematical disciplines as well. First and most obviously, because music was itself a mathematical discipline. This is illustrated authoritatively by Yates, who stresses the Platonic character of these "encyclopaedic" institutions, and finds in them explicit references to mathematics and natural philosophy. Secondly, it is precisely at this time that music was transformed into a much more technical and "mathematized" art. France at the beginning of the sixteenth century was "backward" in comparison with Italy and the German countries.[22] Indeed, the French developed these studies following in the tradition of the Italian Platonic academies, and they also benefitted from the scientific contents of the Italian tradition in music theory. Thirdly, the reform of music theory was connected to language reform, and this in turn was related to the increased influence of some of the mathematical disciplines, such as algebra.

Although not the founder of any of the Académies, Peletier was involved in a couple of them, and while he did not play a major rôle as a participant, he influenced their activities as the initiator of many topics discussed there. But let us first consider the court milieu from which the academies were to arise, and in particular the humanistic circle of Marguerite de Navarre, for this was the intellectual world which Peletier entered after his studies.

Born at Le Mans in 1517, into the family of a barrister, he left for Paris to join the Collège de Navarre where his brother the theologian, Jean Peletier, was the principal. Later, he went back to le Mans and read law for five years. In the late 1530's he became secretary to René du Bellay, bishop of Le Mans. Sometime before 1539 Peletier was introduced to the Platonic circle of Marguerite de Navarre, grandmother of Henri IV.[23]

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22 D. P. Walker. ***Music, Spirit and Language in the Renaissance***.

23 We know this from F. Yates, 1947.

In 1541 Peletier published ***L'Art poëtique d'Horace, traduit en vers François***.[24] In its dedicatory epistle Peletier draws up a manifesto for the French language which inspired Joachim du Bellay in his ***Deffence et illustration de la langue française*** de 1549. Peletier asserts that modern authors are not inferior to ancient ones; only their language is inferior. Therefore new authors must cease trying to write in foreign languages, because one can reach the heights of creation only in ones native language. New authors should, instead, begin to use Greek and Latin only as a source of *inventio* (finding of topics or arguments) and of *dispositio* (arrangement of topics or arguments). This implied that the three other aspects of the art of discourse, starting with *elocutio* (i.e. the choice of words, the style proper[25]), should be in the native language.

In these same years, Peletier developed his connections with the intellectual milieu of the Court. First of all, he participated in the poetic debate. Starting in 1543, he taught poetry to Ronsard and participated in the Pléiade.[26] With Ronsard, he encouraged Joachim du Bellay to compose sonnets and odes.

Later in 1543, Peletier was suddenly appointed principal of the Collège de Bayeux in Paris. This allowed him to participate even more fully in Parisian intellectual life. With his friends the poets he became involved also in the musical and poetic movement of the *vers mesurés*, which were to gather poets around Maurice Scève and Pontus de Tyard. This first

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24 The collection of the *Bibliothèque Nationale* contains the second edition, from 1545. This translation had been reprinted, corrected, in ***Les Oeuvres de Q. Horace Flacce(...)partie traduictes, parties veues et corrigées de nouveau par M. Luc de la Porte, Parisien, Docteur en Droict et Advocat A Paris, 1584.***

25 *Elocutio* was evidently, for Peletier, the most personal and the most crucial part of writing, the one which most specifically involves creation. See Demerson 1984.

26 Yates suggests that he should even be considered a member of it (Yates p. 14). On this question, see also H. Chamard.

informal group provided the example later followed by the more structured Académie de Baïf. The project for both the group and the Académie was fundamentally the same: to write and to perform poetry conceived together with music. This was totally new in the context of contemporary music and poetry, but was conceived of as an *imitatio* of ancient practice in Greece and Rome. This program involved a reform of musical notation,[27] and was accompanied by the campaign for the use of French in literature.[28]

The combination of poetic and scientific interests is apparent already at this early stage of Peletier's career. His second publication is an annotated edition of the ***Arithmeticae practicae methodus facilis*** by Gemma Frisius, (Paris, G. Richard, 1545),[29] which was one of the great successes of its time. Peletier was in fact the commentator on the French edition of this work; the Ramist Forcadel provided a later edition. The author of the Italian edition was particularly interested in reproducing Peletier' contributions.[30] This work is the first of a series of mathematical texts intended to allow the reader to benefit from the good rhetorical (and pedagogical) strategy of the author. Peletier declared in the preface that this text was intended to be useful to the students he trained in elementary arithmetic at the Collège de Bayeux.

In 1547, Michel de Vascosan (Robert Etienne's brother-in-law), together with Gilles

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27 See again the works of D. P. Walker.

28 The poetical side of the question was recently studied by Kees Meerhoff in ***Rhétorique et poétique au XVIeme siècle en France. Ramus, Peletier et les autres***. Leiden, Brill, 1986.

29 A second edition appeared in 1563, with the scientific publisher Guillaume Cavellat: Gemma Frisius. ***Arithmeticae practicae methodus facilis. Cum Jacobi Peleterii Cenomani Annotationibus. Ejusdem item de fractionibus astronomicis compendium... Quibus demum ab eodem Peletario additae sunt radicis utriusque demonstrationes***

30 See Orazio Toscanella's version, published at G. Bariletto, in Venice.

Corrozot, edited ***Les Oeuvres poetiques de Jacques Peletier du Mans***. This marks a moment of more intense interaction between Peletier and his publisher, which was typical of Peletier's career.

### *2. The humanistic program for printing: orthography and the painstaking birth of the scientific book*

Peletier had already published his work ***L'Art poetique d'Horace*** with Vascosan. In it, Peletier announces the introduction of a new orthographic system, which in the text itself is followed only in part, since the publisher did not follow the author's instructions. After this negative experience, Peletier decided to follow the process of printing more closely, and moved to Vascosan's house in the rue St. Jacques. This circumstance, in 1547, coincides with two other developments. In that year, he decided to quit his position at the Collège de Bayeux,[31] and at the same time, Vascosan's house became the center of intellectual life for the Peletier's circle. For, around 1547 Peletier had brought together a sort of philosophical entourage, a circle in which to discuss and develop his program for literary, orthographic and mathematical reform.[32] We find a direct description of it in the introduction to the ***Dialogue de l'Ortografe et Prononciation Françoese*** (Poitiers, J. et E. de Marnef, 1550).[33] The dialogue itself represents a debate at the house of the publisher Vascosan between Peletiers's

31 The reason for this choice will be discussed later. However if, as Catach (1968, p.100) writes, Peletier's main decision was to move to Vascosan's, his quitting the collège was a consequence, because Collèges were strictly residential: see, for instance, Compère 1985.

32 See especially Natalie Zemon Davis *Peletier and Beza part company*, 1964.

33 This text is dedicated to the princess Jeanne de Navarre, grandchild of Marguerite de Navarre. As Peletier explains in the epistle, Marguerite de Navarre had asked Peletier to dedicate such a dialogue to her, but had died in the meantime. It was reprinted in 1555 by Jean de Tournes.

friends, Jean Martin, Théodore de Bèze, Denis Sauvage and Jean Paul Dauron.[34] It should be noticed, parenthetically, that Peletier's program would not be carried out by that same circle. By the time the dialogue was written, the discussion group had already dissolved. A few years later, mostly as a result of religious divisions, only Jean Martin and Jean Paul Dauron would still be counted among Peletier's friends. The contrast between Peletier's and Beza's position can be represented in terms of religious beliefs and thus be characterized as Erasmian versus Protestant.[35] The dialogue was in fact written in Lyon, after the religious wars and the dispersion of his circle made him leave Paris. But let us take Peletier's description as a chronicle of the themes discussed in Paris around 1547. The theme of the work is orthography, the occasion being provided by the publication of Peletier's ***Oeuvres poetiques*** and his great disappointment over his publisher's failure to adhere to his orthographical reforms. Peletier had been interested in this topic from the time when he was secretary to René Du Bellay, around 1530, as he explains to Louis Meigret in the dedicatory epistle. It was in fact an important theme of debate at the time, as, for instance, Meigret's, Sébillet's and Ramus' works show. Only two points about the ***Dialogue*** can be mentioned here. First of all, we note Peletier's insistence on the notion that language comes from the people, and is transmitted by contact with that people. Across time and space, only writing can provide a substitute for the normal transmission of language, which is oral. However, writing is more than a mere substitute for speech. Peletier writes in the ***Dialogue***:

> Voela commant elle ne doet point ètre tant sugette a la prolation qu'a l'antandemant, vu que le plus que nous retirons de l'Ecritture cét l'intelligance

34 At times Conrad Badius and Jean Corbin also participated in the conversation. See Davis 1964.

35 See Davis 1964, p. 210.

du sans.(p. 75)

In this way, Peletier states clearly that writing has autonomy with respect to speaking. But, in particular, writing does not convey pronunciation, and this is why the study and improvement of orthography is so important as a tool for bridging the gap between the written and the spoken language.

The second point is at a different level, and concerns more directly the project which Peletier was to pursue throughout his life, the writing of scientific books in French. Peletier writes:

> Nos mathématiques ne furent jamais mieux au net, qu'elles sont de présent, ni en plus belle disposition d'être entendues en leur perfection. Et par ce que leur vérité est manifeste, infallible et constante, pensez quelle immortalité elles [les mathématiques] pourraient porter à une langue, y étant rédigées en bonne et vraie méthode. Regardons même les Arabes, lesquels encore qu'ils soient reculés de nous et quasi comme en un autre monde: toutefois ils s'en sont trouvés en notre Europe qui ont voulu apprendre le langage, en principale considération pour l'astrologie, et autres choses secrètes qu'ils ont traité en leur vulgaire, combien qu'assez malheureusement. Car on sait quelle sophisterie ils ont mêlée parmi la médecine et les mathématiques mêmes. Et toutefois ils ont rendu leur langue requise en contemplation de cela. Avisons donc à quoi il peut tenir que nous n'en fassions non pas autant,mais sans comparaison plus de la notre? (p. 117-118.)

We shall see in the final chapter that the Arabs could only paradoxically be taken as a model in that context. However, Peletier indicates quite clearly that the plan is to impose French as the language of science or, to paraphrase him, "*rendre notre langue requise en contemplation des sciences*."

Peletier took this program seriously. Not only did he publish an annotated edition of Gemma Frisius' work in 1547, as we have seen, but he wrote his own book on arithmetic, ***L'Arithmetique departie en quatre liures, à Theodore de Beze***.[36] From Peletier's own

36 According to Davis, who used French arithmetic to deal with the changing perception of commercial activities on the part of the aristocracy. See Davis 1960.

words, it seems that this edition of Gemma Frisius was meant to be used in teaching at the colleges. This means that after introducing a new topic at the collège, Peletier introduced it to the Court[37] by means of a newly conceived book of arithmetic in French. While the writing belongs clearly to the Parisian period, it was published after another change in Peletier's life, while he was living in Poitiers at the house of the Marnef, in 1549. We shall see in greater detail, in the next chapters, the innovations that Peletier introduced in his mathematical texts. His influence in the field of commercial arithmetic and algebra was most strongly felt in a separate manual in which he restructured the domain of each of these disciplines. This was new in France. Through changes in the rhetoric of the manuals Peletier made them more acceptable to the wide audience of the court and to the *noblesse de robe* connected with it. In the first phase of the academies, only Platonic arithmetic was considered worthy of study and research.[38] At most, this could be combined with practical geometry, which meant fortifications and *ars militaris*, topics quite appropriate for the *noblesse*. By contrast, for Peletier, practical arithmetic and algebra were seen as legitimate fields of knowledge, representing the good part of commerce. In this he was ahead of his time,since a positive view of commerce was not common before the time of Richelieu.

Here, it is important to stress that he consciously theorized the features of a scientific book. This is, I argue, is an indication of Peletier's increasing interest in the process of printing and the possibilities of dissemination offered by it. A confirmation of this thesis comes from the fact that, once again, Peletier lived at the house of the Marnef family in Poitiers. The Marnef were particularly aware of the ongoing orthographic reform, and ready

37 See Davis 1960.

38 See Davis 1960 as well as 1958.

to be leaders in the field, even competing with the main Parisian scientific publishers, such as Cavellat and Wechel. Thus, they were the main disseminators of Peletier's orthographical reform. In fact, the ***Arithmetique*** is the first book that might have satisfied Peletier from the point of view of fidelity to his orthography. Furthermore, the Marnef household was also a lively intellectual center. Here Peletier saw Elie Vinet, whom he had met in Bordeaux a couple of years earlier.

We have seen that he published the ***Dialogue de l'Ortografe*** in 1550, with the Marnef. Around the end of 1553, Peletier left Poitiers for Lyon, where he lived at the house of Jean I de Tournes. While his position was that of teacher for the publisher's son, Peletier managed to publish a series of works with him, all with carefully reformed orthography: ***L'Algèbre***, in 1554; ***L'Art poëtique*** in 1555, ***L'Amour des Amours*** in 1555; ***In Euclidis Elementa Geometrica Demonstrationum Libri sex*** in 1557; ***Disquisitiones Geometricae*** in 1567, as well as a series of second and third editions of previous works. The relation with Jean de Tournes is the most significant, not only because this house would continue to publish Peletier's works into the seventeenth century, but also because of Peletier's influence on the activity of the house itself.[39] For about five years, until 1558, all the publications of de Tournes were influenced by Peletier's orthography, and in part corrected by him.

Without listing all the places in which Peletier propagated the new creed of reformed orthography,[40] suffice it to stress that the humanists of Bordeaux, and in particular the publisher S. Millanges, were also influenced by Peletier's reform.

I believe that further research on Peletier's activity in connection with publishers

39 I am relying here on Catach 1968, pp.104-107.

40 For this, see again Catach 1968. See also Citton and Wyss, together with Catach's review of this book, Catach 1991.

would reveal more information about that particular kind of orthography which is mathematical notation. Peletier's mathematical works are in fact very well structured from this point of view, and were influential.

In 1555, Peletier published ***L'Art poëtique***,[41] a work in which he systematized a new theory of language, literary creation (*poiesis*), and style, the principles of which were already present in his translation of Horace and other early writings. This work is in fact a *summa* of twenty years of literary study and practice.

It is well known that sixteenth-century humanism involved a profound rethinking of the boundaries and the potentialities of the three first liberal arts: grammar, rhetoric and dialectic. The reform of grammar as a taught discipline had two sides, the definition of ancient language and the definition of vernacular. In fact sixteenth-century France sees a sudden expansion of works on grammar and rhetoric,[42] both in Latin and in French. This was connected with the expansion of colleges and with an increasing demand for manuals, but also with the expansion of printing independently from the colleges, which did not use French. Peletier's early career follows these interests, starting with the translation of Horace, and continuing with a particular aspect of grammar, i.e. orthography. As for the other two arts, Peletier contributed also to *ars poetica*, which although it is generally opposed to rhetoric is actually connected to it. In fact, rhetoric had been classically defined in two ways: as the art of persuasion, according to Aristotle, or as *ars bene dicendi*, by Cicero. Whereas Aristotle's definition connects rhetoric to the art of demonstration, and to dialectic, Cicero's

41 ***L'Art poetique de Jacques Peletier du Mans, Departi an deus livres***. A Lyon. Par Jan de Tournes et Guil. Gazeau, 1555.

42 See the important introductory chapter in Gordon ***Ronsard et la rhétorique*** Geneve, dROZ, 1970.

definition associates rhetoric and poetic art, and stresses the role of rhetoric as a general strategy of thinking, relevant to all fields. Peletier in particular shows the differences between the two while seeing the two arts as essentially connected. Their difference is not reducible, of course, to the choice of verse or prose, for what distinguishes an orator from a poet is that the poet is not limited to a specific subject, and may be as general or abstract as he pleases, whereas the orator must limit himself to particular situations, to cases. Sciences, and particularly mathematics are therefore within the scope of the poet, for what subject is more abstract and general than mathematics?[43]

In Peletier's view *ars poetica*, as well as rhetoric, includes three main parts. These are invention, disposition, and elocution. More precisely, he writes that "*toutes sortes d'écrits s'accomplicent de trois parties principales, qui sont Invention, Disposition, Elocution*." The content of the *ars poetica* itself suggests some rules applicable to writing mathematics, which are in fact applied by Peletier. In this spirit, he insists on poetic quality of clarity for his ***Arithmetique.*** Peletier and his interlocutor Joachim Du Bellay take the thesis from Quintilian according to which clarity is the main quality of a poem. Quintilian wrote "*Nobis prima sit virtus perspicuitas*," and Peletier echoes him: "*La première et la plus digne vertu du poème est la clarté*."

Each chapter of the ***Art poëtique*** is devoted to a rule. Among the most important is the famous one suggested by Quintilian, that of *imitatio*. This concept is common to all sorts of humanistic thinking, but its meaning varies remarkably. It is crucial also for the writing of mathematics, first of all as a properly literary notion, but also because it implies a relationship to ancient sources, including scientific ones.

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43 For this point, as well as for the general situation of French rhetoric, see Gordon and Meerhoff.

Technically, *imitatio* includes various aspects -- elocution, translation, and orthography. In other words, it involves a rethinking of the whole language, with particular attention to the relation between oral and written language, a particular interest of Peletier's. To contribute actively to a written tradition, an undertaking all the more important and rich in consequences in the age of printing, meant to contribute to language. Only two alternatives were possible, either to write in Latin or to transform French. But by the mid-forties it became more and more clear that to write in Latin also implied a choice of audience, and not only a choice of culture. Traditional Latin belonged to the religious tradition and to the University. The humanist reform could at most introduce classical Latin at the university, starting with the Collège Royal. But the nobles used the vernacular, for instance in the academies, as well as the jurists, who developed the *langue du palais*,[44] extending the use of French to high levels of culture. Peletier's reform gave additional impetus to this cultural process already underway. French had to become, like Italian, a classical language. Poliziano and Erasmus' theory of *imitatio* found therefore a new currency in sixteenth-century France. To imitate the classics did not mean to write in Latin but to create a "classical" vernacular. For, France adopted humanist ideas and culture at a time when this culture had developed an internal critique. Hence, classical Greek and Latin were taken as the rhetorical model, but with an already sophisticated notion of model, the one elaborated for the Italian language by Poliziano, on the basis of principles given by Cicero and Tacitus in their relation to Greek "Classical" culture. In the sixteenth century, learning Latin meant learning classical, Ciceronian Latin. But when it came to writing it, or to applying rhetorical rules to the contemporary use of Latin, many questions arose. To

44 See Marc Fumaroli, ***L'age de l'éloquence***, 1980, as well as Jean Céard.

what extent could the rule of imitation of the Ciceronian style, in Latin as well as in the vernacular, be applied? *Rebus mutatis, mutata et oratio*, wrote Erasmus in the ***Ciceronianus***, a work devoted to the *imitatio*. There were two main theoretical turns adopted to elaborate this problem. Erasmus drew subtle new distinctions between Cicero's theoretical discussions and his rhetorical practice, and took Cicero as a model also for his own theory of *imitatio*. In fact, Cicero had faced the same problem when he adopted and imitated Greek culture of the classical and Hellenistic age in order to create a new language and a new classical culture. through translation, adaptation of style, and creation of words. In a different context, analogous questions acquired a new force. It was impossible to know classical Latin phonetics but, as Erasmus had taught, it was possible to arrive at a sound hypothesis, beyond which it was reasonable to accept the inevitable changes over time,[45] Applied to *imitatio* in the vernacular, i.e. in French poetry, this had important consequences for the theory of rhythm or scansion: the quantities of syllables had had one value in Latin and another in French. Similarly, Ciceronian prose had an "oratorian number" which was not to be directly imitated, but adapted to the new language. For these reasons, Cicero, who made classic his own mother tongue, was taken as a model by the partisans of the French language. Besides transposing the rules of quantity, rhyme, and intonation from Latin to French, it was important to translate. Translation was therefore, as it had been for Cicero, a crucial aspect of *imitatio*.[46] In order to make French a classical language it was important to translate the ancient classics, as well as the Italian classics, systematically, keeping in mind

45 See Erasmus ***De pronunciatione***.

46 It would be worth developing this topic, which is discussed at length in the ***Art poëtique***. It is of course of great import for the translation of classical mathematicians, such as Euclid, translated by Peletier. But it would require too many references to contemporary translations of Euclid for us to treat it adequately at this point.

that translation need not be literal to be faithful.[47]

If the first formulations of these theses concerning *imitatio* belong, in their "Northern" form, to the texts of Erasmus and Melanchthon, the author of the *translatio* to French was Peletier.[48] His priority in these matters is clear from his relation to the other two main authors of literary reform: Joachim du Bellay and Ramus. Peletier's influence on Du Bellay and the Pléiade is largely acknowledged. Recently Meerhoff,[49] whose work is devoted to Ramus and his theory of rhetoric, has stated that Ramus depends on Peletier for the whole theory of *imitatio* and language. I think this statement should be strengthened or, simply, that we should draw its consequences by saying that Ramus' theory of rhetoric *and* dialectic was strongly influenced by Peletier's theory of rhetoric poetic. This will take us into a brief digression to compare Ramus with Peletier, a comparison which will become clearer in the fifth chapter.

Meerhoff begins with a critique of Ong's work on Ramus.[50] While acknowledging its merits, Meerhoff also stresses its limits and bias. In particular, Ong attributed to Ramus the transformation of the pedagogical discourse from the oral to the written form, with special reference to the charts and diagrams which had such great success in the German countries and in Britain. Ramism is in fact identified with this later phenomenon more than with the theory of method. According to Ong, this is the key to method, for the passage

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47 See on this crucial topic Glyn P. Norton. "*Fidus interpres*: a philological contribution to the philosophy of translation in Renaissance France", in Cave and Castor 1984 .

48 Other contemporaries, of course, such as Dorat and Lambin, transmitted to France this content of Poliziano's teaching.

49 See Meerhoff 1986.

50 Here I refer to Ong 1958.

from an oral to a written form of teaching would correspond to the dominance of *inventio* and *methodus* (the terminology used by Cicero in the ***Topica***) over other parts of ancient rhetoric. Ong's claim seems to Meerhoff inadequately justified, for Ramus can be mostly be characterized as a teacher at the colleges, and his theory is connected to this fact. In particular, recent studies offer data for a more detailed view of the complex evolution of the Ramist thought.[51] From our perspective, the qualification of Ong's thesis should go as follows: there can be no question that Ramus made full use of the development of printing; nonetheless, his impact should be understood mainly within the limits of Parisian colleges, especially the *Collège Royal*, and then later universities abroad. His conception of rhetoric as *ars disputandi*, for instance, was particularly well suited as a model for those colleges developing all over Europe, including the Jesuit Colleges. Peletier on the other hand, focussed his attention on publication.

Another main difference between the two authors is of course the fact that Ramus adheres to the Reformation and dies for it, whereas Peletier does not. In fact, we have noted that he had been a friend of a major reformer, Théodore de Bèze, with whom he "parted company"[52] as a result of Bèze's religious choices.

In this connection, I would like to add what is, for now, mostly a conjecture: that Peletier was committed to a moderate sceptical program, deriving from his personal interpretation of Cicero. This philosophical commitment perhaps adds another layer of motivation, along with the religious reasons already explored by historians, for some aspects

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51 See Nelly Bruyère ***Méthode et dialectique dans l'oeuvre de La Ramée***. Paris ..., as well as ***Pierre de La Ramée (Ramus)***, in ***Revue des sciences philosophiques et théologiques***. Tome 70 n.1 1986.

52 See Davis 1964.

of Peletier's behavior. Peletier's philosophical standpoint can be more precisely described with reference to the interpretations of Cicero current at the time. Erasmus and Melanchthon had in fact provided the two major interpretations, the influence of which was increased by their pedagogical character. However, new editions and translations of Cicero contributed to the definition of the Latin author independently from any religious orthodoxy; they stressed instead his skepticism and epicurianism. It seems that in sixteenth-century France this line of interpretation was more easily accepted by Catholics than by Protestants, both because the protestants had a new orthodoxy to defend and because of their theoretical proximity with St. Augustine, who had provided the main Christian interpretation of Cicero. The counter-reformation Catholics would of course take a different view, but this does not affect the case of Peletier.

This pagan interpretation of Cicero arrived in sixteenth-century Paris at the same time as the diffusion of interpretations in terms of dialectic, such as those propounded by Lorenzo Valla and Rudolph Agricola.[53] It combined with the campaign for a national language and the expansion of printing. Many elements of the philological interpretation of Cicero and of the dialecticians were shared by all "Ciceronians" in sixteenth-century Paris, a communality of belief which oriented the foundation of the Collège Royal. However, a more specifically skeptical and scientifically oriented brand of Ciceronianism was typical of the Parisian group connected with the academies, which, far from disappearing during the religious struggles, continued through the St. Barthelémy massacre. Already Yates made some remarks in this direction,[54] indicating that much academic Pythagorism came from

53 See in particular Lisa Jardine 1983.

54 See for instance op. cit. p.39 on the *Timaeus*, or pp. 82 and 87.

Cicero, or was at least to be found in Ciceronian interpretation. More recently Charles Schmitt has specifically studied this Parisian intellectual context as Ciceronian skepticism.[55] He devotes a few pages to Guy de Bruès, author of ***Les dialogues contre les nouveaux academiciens*** (1557). This work is particularly important for determining Peletier's intellectual context because it represents two members of the Pléiade as opponents in the dialogue, Jean Antoine de Baïf as a skeptic and Pierre de Ronsard as anti-skeptic. The subject matter of the debate was provided by Cicero's ***Academica***. It is certain that Peletier was involved, at least indirectly, in the debate occurring in real life within the Pléiade. We can take as a working hypothesis that even a weak commitment on this point had something to do with his indefinite position in religion and especially with his distance from his friends of the rue St. Jacques, after some of them adhered to the Reformation. This hypothesis accords with certain facts, such as Peletier's friendship with Montaigne and Pontus de Tyard. For Montaigne writes[56] as does Pontus de Tyard, about providing hospitality to Peletier. Tyard gives also the date of 1558, after the publication of the Euclid and preparation of the Latin edition of the algebra. Furthermore, we know that Peletier wrote a manuscript on scepticim, described as "réfutation du pyrronisme" by Laumonier.[57]

Yates' and Schmitt's studies, along with Peletier's mathematical writings themselves, encourage us to see his project in Ciceronian-skeptical terms, independently from Ramus' views, but with some convergence because both took as a common source Cicero's approach

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55 See Charles B. Schmitt. ***Cicero scepticus. A Study of the Influence of the 'Academica' in the Renaissance***. The Hague, M. Nijhoff, 1973: chapter IV, "The *Academica* at Paris in the middle of the sixteenth century: Talon, Galland and others."

56 See ***Essais***, p. 324, where he describes a conversation on what appear to be asymptotes, which Montaigne takes as a limit to reason in the sciences.

57 Unfortunately, I have been unable to trace, much less read, this manuscript.

to culture (the liberal arts) and to science, language, and religion. Once again, the key here is the notion of *imitatio*.

The main point of commonality between Ramus and Peletier is their relation to the colleges. For this reason, we shall start our discussion of Peletier's connection with the colleges with a comparison with Ramus.

### *3. Peletier and the Colleges*

Both Peletier and Ramus studied at the collège de Navarre in Paris, where Oronce Fine taught mathematics. Peletier and Ramus shared many interests and theoretical theses, both working on grammar, orthography, the geometry of Euclid, algebra, and rhetorical tradition. While it seems now that on all these topics Peletier preceded Ramus, Ramus had an enormous impact on education, within France as well as abroad.

Jacques Peletier du Mans never identified with the role of professor in Paris, even though, in his last years, he became more oriented toward education.[58] However, he started and ended his career directing a college at the University of Paris. His interest in education was therefore not only a part of his humanist education, but also an aspect of his practical activity. Since he was not a *lecteur royal*, he was not asked, as was Ramus, to draw up teaching programs. Nonetheless, he contributed to changes in the topics and style of teaching through his publications.

Ramus had much more influence on the colleges and on generations of teachers, while Peletier, instead, aimed his reform at the intellectual milieu of the academies and, even more, at the public he could reach through publication. Improving the use of printing was in

58 See Jugé, p. 60ff.

fact his main goal.  Accordingly, he published many different sorts of works: translations of classics, poetry, mathematical and medical theoretical treatises, scientific poetry, manuals of practical geometry and astrology, epistles.  Experimenting with these various possibilities seems to have been one of his main goals.  Even though a part of these works was addressed to teaching, they were newly conceived for improving the *genres* to which they belonged, and developed their topics with much greater attention to the reader than Ramus' works. The disciplines were treated in detail, not merely with a list of topics and a discussion of the sources, full of digressions, as in Ramus's works.

To sharpen the contrast between the two authors, let us take a passage from the specific discussion devoted by Peletier to the question of how to write mathematics.  Peletier deals with this question in the `proème' of the second book of his ***Arithmétique***:

> Entre les hommes d'érudition, ami Debeze, a été longuement debatu, et n'est encore le differend vidé, lequel des deux est le plus profitable pour l'entretenement des arts et disciplines, que les professeurs d'icelles, quant ils les mettent par écrit, les traitent clairement et au long, ou bien obscurement et brief. (f.XXVI)

Peletier tries to give his own answer.  Some authors were in favour of a pedagogy which did not simplify the topic to be treated, for simplification stops curiosity.  Such a reason was not without force, especially at the moment of expansion of printing, a technological watershed comparable only to the age of television:

> Et qu'ainsi soit, disent ils, depuis l'art d'Imprimerie inventé, on n'a point veu de personnages de savoir en si grand nombre ni de telle solidité comme on faisoit au temps passé, parce que les hommes aians multitude de livres a commandement, veulent embrasser non seulement plusieurs auteurs d'une profession, mais plusieurs professions diverses: qui est cause qu'en se chargeant l'esprit de tant de choses, ils sont contrains de laisser de chacune une grande partie par les chemins, et se trouvent enfin frustrés de toutes. (f.XXVII)

But Peletier raises two substantial objections to this point of view; first, that dispersion should be corrected in the readers, not in the texts and second, that new technologies also provide new methods of learning. He writes:

> A la verité nous voyons qu'aujourd'hui on a trouvé moien d'abbreger le temps aux disciplines par clairté et facile maniere d'enseigner. Comme on peut voir de la Grammaire, Retorique, Musique et autres professions.(ib.)

The solution chosen by Peletier tries to take into account the two extremes:

> Car apres avoir examiné le merite des deux contraires, je trouve qu'il n'est pas impossible d'être facile et brief tout enseimble, pourvu qu'on tiegne tousiours son addresse a la metode, qui est celle qui donne majesté aux écriz, et non l'obscurité.(ib.)

But in the specific case of mathematics this is a particular challenge:

> ...qu'il faut confesser qu'en matière de Mathematiques quelque metode qu'on tiegne, et quelque lumiere qu'on leur puisse donner, si sont elles tousiours difficiles quelques peu, au regard des autres professions. Car qu'elles soint si difficiles d'elles mesmes, c'est plus une opinion de credit que d'experience."(f.XXVIII)

This was been written about the ***Arithmetique***. And Peletier did in fact succeed in his program to popularize mathematics in the French language, to judge from the number of editions and from annotations in the extant copies.

Now we can see also a more technical point of difference between the philosophical and rhetorical views of mathematics. Clarity, that most important feature of a poem (for Horace) and thus of a scientific text (for Peletier), was also, together with utility, the main feature of mathematics for Ramus. In fact, the second and third books of the ***Scholae mathematicae*** (1569) are devoted to the problems of *inutilitas* and *obscuritas* of mathematics. This was in itself a *topos*, of course, about mathematics, and particularly about mathematical teaching and learning. But it seems that while *utilitas* concerned Ramus alone, *clarity* concerned both authors, but with a strong emphasis on publishing for Peletier and a strong emphasis on pedagogy for Ramus. In fact, Ramus appears much the more conventional of the two in his presentation of *topoi*. While Peletier also goes through the classical steps in writing his text and in explaining the criteria used, he shows a great awareness of the potential of printing and publication. It is not surprising that Ramus wrote textbooks in Latin, while Peletier wrote in French.

In fact, as we know, not all of Peletier's works are in French. After the years in which he followed his publishers, came the years in which he simply wrote in Latin. This has led historians to think the change was due to a disenchantment with French publishing and French orthography. This was certainly part of Peletier's attitude, but I want to stress that this was also part of the process of putting to use Peletier's teaching in the colleges.

After his stay at Jean de Tournes' in Lyon, Peletier went to Savoy as an expert in *ars militaris* and doctor of medicine for the Maréchal de France Charles de Cossé-Brissac, lieutenant général du Roi en Piémont. It seems that he was also the mathematics instructor for Brissac's son. This is the time in which he dedicates its ***L'Algèbre*** to Cossé-Brissac. In the meantime, Jean de Tournes published Peletier's Euclid. For, when he was at Jean de Tournes', Peletier was engaged to teach mathematics to the publisher's son, and he did so by using Theon's version of Euclid, i.e. a version closer to algebraic interpretations.[59] His experience of private teaching for Jean II de Tournes contributed, it seems, to Peletier's writing of ***In Euclidis Elementa geometrica Demonstrationum libri sex***, a work which was to be followed by other publications in the field of geometry. They are, of course, in many ways, related to Peletier's his arithmetical and algebraic writings. Unfortunately, we cannot

59 We have his witness in the seventeenth-century edition of Peletier's Euclid.

give it the space it deserves, beyond mentioning that it was certainly an element of his work connected to teaching in the colleges.

In this connection, it should be noticed that, in accord with his program of "vernacularization," the humanism of Peletier is not that of the classicists. For a translator of Euclid, it is astonishing to see that he employs hardly any classical references in a "gratuitus" way. Peletier banishes the classics by choice, and not only because his books are meant for practical use. By contrast, Stifel had introduced a section on the tenth book of Euclid's ***Elements***. We need not mention the contemporary Buteo, who traces algebra itself to Euclid, or Nunez (and eventually Tartaglia), who while writing in the vernacular demonstrates the formulas of some equations by means of geometrical demonstrations. After Buteo's attacks on Peletier's Euclid, Peletier even engages openly in polemics against Buteo, focusing precisely on his "Hellenisms," beginning with the denomination *logistica*. Peletier, by contrast, orients all the force of his writing towards the learning of a technique, and an adequation between the theory and the practice of the art.

Margolin[60] states that the choice to write in Latin was in this case due to the insistence on the scholars' part (and criticism about his peculiar orthography) and the hope of acquiring a larger audience abroad. Moreover, it should be stressed, once again that Latin was the language of the Colleges, and the first six books of Euclid were the typical instruction in mathematics at the colleges. By contrast, French was the language of the academies, that is, the military colleges. To this kind of institution[61] seems to be dedicated the late translation *into French* of this same work, published by Jean II de Tournes in 1611

60 See Margolin 1976, p.120.

61 See on this kind of school Chartier, Julia, Compère.

in Geneva.

Between 1557 and 1569, Peletier is in Paris, revising his works. In particular there is a work of the earlier years, the first edition of ***L'Algèbre***, which is a clear example of the effort to write science in French. Yet, the second edition of this work is in Latin.[62]

Historians have made many hypotheses as to what this indicates about Peletier's choices. In his preface to the translation of his own ***l'Algèbre*** he affirms his desire to explain algebra because he is in the course of working on the tenth book of Euclid. But there is no significant change between the French and Latin versions. Verdonk[63] sees the reason for this translation in the difficulties of having printed works which employ the new orthography. Margolin[64] perceives, moreover, a transformation in Peletier's work from the period in which he insists on writing in French to the period (from 1560 to his death in 1582) in which he resigns himself to writing in Latin. The public had not necessarily changed. What had changed was the atmosphere, now dense with religious and political conflicts. There were personal conflicts as well, for Peletier had to defend his candidacy at the Collège Royal and his theory of angles of contact. I agree with Margolin, but I also think that Peletier's pedagogical project had been left behind by changing times. From the grand moment when the project was first unfurled, there was a steady process of institutionalization, for we know that between 1560 and 1600 more than a third of the

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62 ***De occulta parte numerorum***, Paris, Cavellat, 1560. Margolin 1976 (p.118) writes: "Pontus de Tyard, dans le discours du *Premier Curieux* fait allusion en mai 1557 au travail de Peletier, qui 'revoyait son Algèbre pour la donner aux Latins'."

63 See Verdonk 1966.

64 Margolin J.C. "L'Enseignement des mathématiques en France (1540-70). Charles de Bovelles, Fine, Peletier, Ramus", in ***French Renaissance Studies***. ed. P. Sharratt. Edinburgh, 1976.

colleges in existence at the time of the Revolution were founded.[65] It may be the case that the project concerning language, and in particular the effort to switch mathematics into the French vernacular, had come to a halt, if only temporarily, as the subject was increasingly taught by the colleges. It is enough to consider some of the Jesuit colleges, where, as Dainville has shown, the teaching of mathematics in Latin gained importance. Here, I take the Jesuit colleges as a paradigm for developments in the study of mathematics; but they are also a model for the institutionalization of humanist projects for learning, e.g., history and logic. Properly understood, the same comes to pass in the Protestant camp, as one can ascertain in the teaching of the mathematician Dasypodius at Strasburg. To return to Peletier, it seems that in this second phase, which is to say after 1560, his ambitions of advancing a politics of culture must have lost momentum. This despite the fact that the project of moving mathematics into the French language continued to be carried out little by little. So, we see his works being published in both languages.

In fact, we seem to have a proof of the use of Peletier's text in the Colleges. A very interesting copy of this edition ***De occulta parte numerorum*** belonged to Henri de Monantheuil, *lecteur royal dès mathématiques* de 1573 à 1606. It is abundantly annotated.

Furthermore, it seems that this edition had a particular *fortuna.* Of special interest for us are the two copies of ***De occulta parte numerorum*** in the library of the Sorbonne, which belonged to Kenelm Digby, and another copy which belonged to Jacques Alexandre le Tenneur.

Finally, two facts about this book should be mentioned, first, that it contains an

65 Concerning the Collège Royal and the pedagogical transformation in the strictest sense in the parisian colleges during the 1630's, see Roger Chartier, Marie-Madeleine Compère, Dominique Julia. ***L'éducation en France du XVIe au XVIIIe siècle*** (Paris, Société d'Edition d'énseignement Supérieur).

epistle by Peletier *ad Razallium*, and second, that it was published by Guillaume Cavellat, who was responsible for many scientific publications in Paris at the time. He had published the algebraic book by Scheubel[66] at the request of Jean Magnien, lecteur royal ès mathématiques between 1555 and 1556, who wanted to use it in his classes. Cavellat was well aware of the need for mathematical books newly conceived and for a new care in mathematical notation.[67] In this sense he shared Peletier's outlook. In fact, their collaboration had started with Gemma Frisius's edition. Finally, it should be said that at the end of the epistle, Peletier declares "Totus sum in medicina." In fact, these are also years devoted to the study of medicine, and he seems to have connections to some Parisian doctors, since he dedicates his work to Johannes Capellanus, Regis Archiatrum.

During these years he might still have seen some of the members of the Académie de Baïf, such as Etienne Pasquier. In fact, Etienne Pasquier wrote about many participants in the Académie in his ***Poemata***, published at Gilles Beys. He wrote also about Peletier:

> Jacques Peletier du Mans commença d'habiller notre Poésie à la nouvelle guise avec qu'un tres heureux succes... Ce fut un belle guerre qu'on entreprit alors contre l'ignorance, dont j'attribue l'avant-garde à Scève, Beze et Peletier; ou si vous le voulez autrement, ce furent les avant-coureurs des autres poetes.

Here poetry is clearly meant to encompass science.

In 1563 Peletier publishes in Basel ***Commentarii tres, primus de dimensione circuli, secundus de contactu linearum, tertius de constitutione horoscopi***.[68] In 1567 he publishes his ***Disquisitiones geometricae*** (Lyon, Jean de Tournes). In 1572 he goes back to

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66 ***Algebrae compaendiosa facilisque descriptio***, Paris, 1559,

67 This is well demonstrated by Isabelle Pantin in ***Imprimeurs et Libraires Parisiens***, 1986.

68 There will be a separate edition of the second essay, published in Paris in 1581.

Savoy, to the entourage of Marguerite de France. But he manages to publish another important geometrical work, ***De usu Geometriae liber unus***. Paris, Gourbin, 1572. It is immediately (in 1573) printed in French also as ***De l'usage de la Geometrie*** by the same publisher. In 1571 Peletier might have met Viète, who was in Paris for the first time between 1571 to 1573.

In 1572 Peletier went back to Bordeaux and directed the Collège de Guyenne, the collège of Elie Vinet, but which was now in its decline. He remained there for a few years, detained by the religious struggles.

In 1579 he taught mathematics at the Université of Poitiers. In 1580, Peletier was back in Paris. His presence in Paris allowed him to pursue his connections with other mathematicians. Among these were probably Gosselin and Viète, Gosselin as a participant to the Académie de Baïf, and Viète insofar as he was in Paris for the second of his three sojourns from 158O to 1584.

Less satisfactory was probably his involvement in the dispute about the angles of contact. This brought him into conflict with another *lecteur royal*, Henri de Monantheuil, and with Clavius ***In Christophorum Clavium de Contactu Linearum Apologia. Eiusdem Demonstrationes tres: 1 De Anguli Rectilinei et Curvilinei aequalitate. II De lineae rectae in treis parteis continue proportionales sectione. III De arcae Trianguli ex numeris aestimatione***. Paris, Marnef, 1579. This might have been one of the reasons why Peletier's works are not easy to find in the libraries of Jesuit colleges.

His fame as a mathematician (or at least as a philologist of mathematical texts) must have been great, for it placed him among the select few who could aspire to the succession of the Ramus chair of mathematics at the Collège Royal. This much is clear from the charges levelled by Bressius (Maurice Bressieu, "lecteur royal" between 1575 and 16O8, and the first successor of Ramus), against Peletier and to which Peletier responded in his ***In M. Bressium Apologia***. Paris, G. Richer, 1580. This able defense of his position seems to have facilitated his access to the rôle of principal at the Collège du Mans, in Paris. There he died in 1592, after the publication of a new edition of his ***Oeuvres poëtiques*** Paris, Rob. Coulombel, 1581.

Peletier's life has appeared too chaotic to many biographers. I think that his peregrinations are to be ascribed to his attempts to avoid being involved in academic as well as religious struggles:

> Nunquam equidem fore existimavi ut de meae vitae rationibus cum ullo hominum esset disceptandum. Sic enim me in omne vita gessi ut meae conscientiae praesidio innixus, hominum opinioni parum servierim (***In M. Bressium Apologia***, fol. 12 recto)

The other major element, the encyclopedic knowledge and range of publication, is not surprising, because quite a few great people of the time possessed this talent and this ideal. But Peletier had theorized this aspect of his nature in his program for the poet, a program he was, himself, able to fulfil:

> L'ofice d'un Poëte èt de donner nouueaute aus choses vieilhes, autorite aus nouueles, beaute aus rudes, lumiere aus obscures, foe aus douteuses, e a toutes leur naturel, e a leur naturel toutes.(***L'Art poëtique***, p.24)

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***Part B: Guillaume Gosselin and the conception of algebra as a discipline: the second phase of the French algebraic tradition***

Guillaume Gosselin is not a well-known figure in the history of France, although he was renowned in his time. For while, he was considered among the main French algebrists, for instance by Italian authors up to the mid-seventeenth century, and by Leibniz. Here we shall summarize what is known about him,[69] adding information which derives from this research.

We do not know Gosselin's date of birth, but he was apparently young at the time of the publication of his work ***L'Arithmétique de Nicolas Tartaglia Brescian***. Paris, G. Beys, 1578, because one of the poems accompanying the text is dedicated *Ad Gullielmum Gosselinum iuvenem studiosissimum Mathematices*. While this does not tell us how young he actually was, it does suggest that he was perhaps still under 30. We know the approximate date of his death from Bachet de Méziriac, who edited the definitive edition of Diophantus in 1621.[70] Bachet writes in the introduction that he died of the plague, probably in Paris, around 1590.

From the text we know that he was resident at the Collège de Cambrai. We do not know what his status there was but, since he was the author of a book, he probably held a teaching position, as a *regens* or a *submonitor*.

Gosselin does refer to the instruction of young people in mathematics in his first work, ***De Arte magna seu de occulta parte numerorum quae algebra et almucabala vulgo dicitur***. Paris, G. Beys, 1577, in the dedicatory letter. This pedagogical aim is declared also

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69 La Croix du Maine - Du Verdier is not of great help here. More informed is the ***Biographie Française*** and Goujet. Even Montucla makes some mistakes in attributing works to Gosselin.

70 See Diophantus, a Bacheto.

in the third of his works, a *praelectio*, ***De ratione discendae docendaeque mathematices***. A *praelectio* was usually a sort of "syllabus" for a course. This work was printed in 1583 in a gift edition with no indication of the publisher.

His role at the Collège de Cambrai is possibly one of the reasons why he published both of his works at Gilles Beys, who was Imprimeur juré de l'Université de Paris.

Gosselin flourished between 1577 and 1583, living out his short but very prestigious career in direct connection with the Parisian scientific milieux. In this second phase of the French algebraic tradition, we can distinguish three of them -- first of all the Court and the Académie de Baïf, secondly the colleges where algebra was taught, and finally the Parlement with its high magistrates. These distinctions of course should not be misleading. In fact. many of the people involved participated in all three circles.

### *1. The Court and the Académie de Baïf*

Guillame Gosselin was born in Caen. Even though he might have received his higher education there, we find him in Paris and very well-connected at the Court at a young age. This was, probably, in part due to the fact of being a relative of the older Jean Gosselin, author of a few publications on calendar reform and music, and mostly known as an astrologer. He was in charge of the Bibliothèque du Roi, and court mathematician for Marguerite de France, queen of Navarre.[71] We know this connection directly from Guillaume Gosselin, whose translation of Tartaglia is dedicated to Marguerite de France. In the dedicatory epistle, Gosselin emphasizes his long-standing connection to the court and he reminds Marguerite of his relation to the older Jean Gosselin.

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71 Sister of Henri III, wife of Henri IV.

As with the court, Gosselin's connection to the académie de Baïf is made clear in the front matter of his version of Tartaglia. The work is prefaced by a few poems, one of which is by Antoine de Baïf himself. The author stresses the importance of mathematics and the fact that is is founded on a faculty typical of human nature. The way in which this apparently bland thesis is stated indicates that it should interpreted as a skeptical assertion of the mitigated sort. If this is not surprising for Baïf, it might indicate that Gosselin shared this viewpoint. Another dedicatory poem is by Jean Dorat, one of the main characters in the life of the academies. Certainly, this is not necessarily a sign of special connection between the poet Dorat and the author. It does, however, show that the publication of a book by Gosselin was an event of some importance. For, as Yates pointed out, "there was hardly a book of importance published in France in the latter part of the sixteenth century which the great teacher (Dorat) did not encourage with a set of congratulatory verses."[72]

According to Natalie Davis,[73] Gosselin was one of the mathematics "teachers" at the Académie de Baïf. Her thesis is an application of Yates' idea that this Académie focused on all mathematical sciences. Where Yates had complained that for lack of documents and in particular of exhaustive lists of participants like the Livre de l'Académie, we could not know the names of the scientists participating in the Académie de Baïf, Davis supplies documents for a small number of cases. To Davis' evidence we may now add some details confirming Gosselin's connection to the Académie de Baïf and to the legal élite present at the court.

First of all, Gosselin was the object of a poem by Jacques Courtin de Cissé. The poem belongs to a collection called ***Les oeuvres poetiques de Jacques Courtin de Cissé***,

72 See Yates, p. 14.

73 See Davis 1958.

published in Paris in 1581, by Gilles Beys. Cissé's collection is devoted mainly to praise of members of the Académie de Baïf. It included a series of *Odes,* for instance, an epithalame à Anne de Joyeuse and a poem honoring Claude Binet. The first of these was among the founders of the Académie de Baïf, and on the occasion of his marriage in 1581 we know that the Académie organized a memorable celebration. The second was one of the prominent members of the Académie de Baïf. In the "ode" to Gosselin, Cissé invited him, one would hope rhetorically, to abandon "ton subtil Diophante" in order to devote himself to more noble reading. Gosselin's presence in this company shows us without a doubt that he shared membership in the Académie de Baïf and suggests that his role was that of the mathematician. But the book implies an exchange: those to whom the *Odes* are addressed are also the authors of other poems in the same collection. In fact, Cissé stresses Gosselin's poetical talent, of which two poems in the collection give us a taste. Here, he signs himself "Guil. Gosselinus Issaeus", as in the *praelectio*, whereas in the other two works the designation of his origin is "Cadomensis", i.e. "de Caen." The first of Gosselin's poems is in Latin, the second in French. The first consists only of two distichs in honor of Cissé, with the title "In amores Iac. Cortini Cissaei":

> Mittitur ex diva superum Cortinus arce,
> Cum rapido tellus finditur igne Canis,
> Candida cumque albis dominatur stella quadrigis
> Quo caleat Phoebo, quo caleat Venere.

(***Les Oeuvres poetiques***, p.iv v)

The second is "Sur les amours de Rosine a M. de Cissé":

> Venus un iour de Lay son Adonis cherchoit
> Quand du Ciel ici bas elle veit une rose
> Quand sur le sein vermeile de cete fleur éclose
> En memorie de lui des pleurs elle versoit.

Phoebus sur l'horizon à peine se haussoit,
Donnant par sa chaleur l'esprit à toute chose.
Qu'il feist, nouveau miracle, une metamorphose.
De larmes, et de fleurs, don ROSINE naissoit.
Pour cueillir cete fleur tu touchas son rameau,
Separant le saint germe au cieux de l'arbrisseau.
Rechangea tout soudain son naturel pouvoir,
Pour une mi-Deesse, et Fille conçevoir.

II

Venus s'en aperceut, qui du fait indignée
Malgré tous tes efforts te vint charmer les yeux,
Et versant dan ton cueur son poison amoureux
T'esclava sous les loix de cette Rose-née.
Encore elle planta sur ton front to idée.
Si tu ne la réforme au vray patron des Dieux,
Combien triste pour toy sera cete iournée.
Mais quoy? ne pourrois tu sa forme rechanger
En entiere Deesse. On dit que le Poëte
A tout ainsi qu'un Dieu sa Nature parfaite.

Courage, ie te voy desia nouveau Berger,
Ta ROSINE, Cypris protée sur les ailes
De tes vers tous-divins au rang des immortelles.

This book was followed by another text showing Gosselin's participation in the Académie, the translation of Synesius of Cyrene, again by Courtin de Cissé, which is dedicated to Henri III. Some of the previous names appear again as authors, such as Claude Binet. Besides Gosselin, we have also the mathematician Miles de Norry.[74] Gosselin's contribution is the following poem:

Voici l'Hercul', qui de son bras guerrier
Domte le vice, et qui loin du vulgaire
Guinde son vol par un nouveau sentier
Pour doucement tromper notre misere:

74 See Davis 1959 and 1960.

Voici l'Hercul, qui brave avanturier
Quitte la Grece, et se donnant carriere
Iusques en France, a traversé la mere.

Pour nous ravir de sa sainte priere
Accompagné de son Chrestien Mercure
Il rangera cete humaine nature,
Qui, miserable, auroit laisse son Dieu.

Puis comme Hercul s'est assis tout en flame:
son cors tout pur, toute pure son ame
tourneront chercher leur premier lieu.

Actually, Gosselin also included one of his poems at the beginning of the most elaborate of his own publications, the translation of Tartaglia. He writes:

Cuncta Deus fecit numero vel pondere, Cuncta
Mensura,sanctis condidit atque modis
Iamiam quae Latios docuit Tartaglia, et artem
Horum praeclaro dogmate constituit.
Sed nunc Gallus adest, Gallus Latiumque recantat,
Italiam linquit, numeris ut Gallia fiat
Splendidior, tanto nobilitata viro.
Ergo Tartaleas placeat, relegatur, ametur
Omnibus atque modis qui dedit arte modos.

We can see from the two last poems, one describing the translator Courtin de Cissé, the other dealing with the translation of Tartaglia, that Gosselin is concerned with the same problem of cultural transmission which we have seen in Peletier. Greek culture and, in the latter case, Italian culture (what God taught the Romans, here called Latios), needs to be transferred into France. Courtin as Hercule recalls the idea of the "Hercule Gaulois", used by Lefèvre de la Boderie, as we shall see in the sixth chapter.

***2. The jurists. Gosselin's edition of Diophantus***

As for Gosselin's connection with the legal élite, we should recall first of all that *de arte magna* was dedicated to Renaud de Beaune, maître des requêtes at the parliament of Paris as well as "conseiller d'état." Gosselin's connection to him could have been determined by the fact that both were connected to the Court. Very influential at the court from the beginning, Renaud de Beaune is considered to have determined Henri IV's conversion to Catholicism. He was also connected to Henri III's Academy.

An important link to the world of the jurists could have been, for Gosselin, the acquaintance with Maurice Bressieu, who belonged to a family of magistrates, whom we have seen in competition with Peletier for the chair of *lecteur royal ès mathématiques*. Born in 1546 en Isère, he was probably of Gosselin's generation, and they were obviously friends, or so they appear on paper. Perhaps the first published mention of Guillaume Gosselin is contained in a text by Maurice Bressieu1. In his ***Oratio de mathematica professione a P. Ramo instituta et ab amplissimo senatu confirmata***, pronounced at the time of his election in 1576 but published a year later by G. Gorbin, Bressieu mentions a few people among his acquaintancies who are relevant for mathematics. Among these, we find Candalle (the famous translator of Euclid), Peletier, and Gosselin.

From secondary sources,[75] we know that in 1576 he followed Cujas' lectures, and was connected to Ronsard and the président de Thou. Furthermore, the first two *orationes* are dedicated, respectively, to Auguste de Thou and to Barnabé Brisson. We know also that between 1584 and 1586 Bressieu lived at the house of de Thou, and studied Euclid's

75 ***Dictionnaire de biographie française***.

***Elements***[76] with him. In the lecture dedicated to de Thou, Bressieu mentions Gosselin as a "clien" de Thou. It should be noticed that Barnabé Brisson was connected to Viète.

Gosselin, in the dedicatory letter to de Beaune in 1577, calls Bressieu *fortissimus Athleta, cuius tum in graecis tum in mathematicis valet authoritas*. Gosselin writes that Bressius is in charge of the edition of Heron's works, available at the Bibliothèque du Roi.

In fact, from this passage it appears that the Bibliothèque du Roi was accessible to Guillaume Gosselin in spite of the fact that his relative Jean Gosselin, the librarian, was famous for preventing anyone from seeing the books. It seems even that there was a large project underway, of which de Thou and de Beaune were patrons, to edit the main classical works in mathematics.

Gosselin declares that he has written a work in which he has solved many important problems from Diophantus, for instance the ones involving "fictitious" equations *et alia multa ab Interprete non plane intellecta* but that he will wait to publish it until the other books available at the Bibliothèque du Roi appear (*si non omnes, certe quos hucusque desideravimus, qui extant in Bibliotheca Regia)*. This statement suggests, but does not prove, a number of things -- that he worked on Xylander's translation (1575), though it could also refer to Planudes; that he had prepared a commentary, or maybe an edition; that he was waiting to consult the text in the Bibliothèque du Roi or that he was waiting for an edition,[77] but in any case that the Bibliothèque du Roi contained a complete manuscript of the thirteen books of the ***Arithmetic***.

But in the preface to his later work, his *praelectio* ***De Ratione***, which, to my

76 See Merez, ***Vie de M. Bressieu***. 1880.

77 By Bressius, as Bosmans and then Ver Ecke, drawing from Bosmans, maintains. Therefore, they believe that Gosselin was not in charge of an edition, but of a commentary.

knowledge, has never been studied, Gosselin gives more information about his connections to the legal élite, his projects in the recovery of Diophantus, and his patrons.

In the dedicatory epistle,[78] Gosselin addresses Jean de Chandon and Charles Bocher directly with the title "maîtres des requêtes", explaining how he came by the idea of writing a course in mathematics. At the end of the work, Gosselin goes on to say that he is going to produce a book on Diophantus' ***Arithmetic*** and acknowledges that he has been entrusted with this special task by Viète, Cujas, and Holler.[79] Here it really seems that his book should consist of an edition, given that he writes that he is working on the thirteen books of Diophantus, and that the text is full of mistakes, and that those patrons were confident he could correct them. We should notice that, after six years, his and Bressius' plans could have changed, and could have evolved from a commentary to an edition.

What this text does tell us is that the various major figures in the Parlement were Gosselin's patrons. *De ratione* was obviously aimed at these five men, but its wider audience has not yet been established. So far this is the only proof of the connection between Gosselin and Viète. Furthermore, this particular connection is highly meaningful, because Viète was to be a patron for the edition of Diophantus, Given that both authors consider Diophantus the Greek authority for algebra, and the discovery of the work as the main matter for algebraic research at the time, we have here the first trace of the fact that Viète had a Parisian interlocutor on algebra in 1583, i.e. eight years before the ***Isagoge***.

Unfortunately, Gosselin's other work does not clarify matters. Gosselin had clearly

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78 For the text and the translation see chapter 4.

79 Jacob Holler was a jurist and a parlementaire. He is mentioned as a "doyen" in 1546 in a text quoted by Sedillot 1869.

read Diophantus before writing the translation of Tartaglia,[80] and his annotations take some points and problems of Diophantus into account. But especially in his ***De arte magna*** some of Diophantus' problems and techniques are presented and explained at length. Gosselin announced in the introduction to his Tartaglia that he was going to write on Diophantus:

> ie te feray part en bref d'autres miennes veilles sur l'autre partie des nombres, qu'on appelle Algebre, et te rendray Diophante facile, en restituant ce que l'Interprete n'a point entendu.

Thus, while this description could even correspond to the book on algebra, ***De Arte magna***, itself, the date suggests again the composition of a commentary or an edition as announced in ***De Arte magna***.

This project is further, if tragically, confirmed in a story from Bachet de Méziriac, that Gosselin obtained a copy of the Vatican manuscript of Diophantus from Jacques Davy du Perron, but that it disappeared after Gosselin died of the plague around 1588. Incidentally, Davy du Perron, clearly a close associate of Gosselin, and another prominent member of the Académie de Baïf, provides another link between it and Gosselin. He also worked closely with both Gosselin and Peletier and supported both of them. Bachet writes at the beginning of his edition:

> Ioannes tamen Regiomontanus, tredecim Diophanti libros se alicubi videre asseverat et Illustrissimus Cardinalis Perronius, quem nuper extinctum magnum Christianae et literariae Reipublicae detrimento conquerimur, mihi saepe testatus est se codicem manuscriptum habuisse, qui tredecim Diophanti libros integros contineret, quem cum Gulielmo Gosselino concivi suo, qui in Diophantum commentaria meditabatur, perhumaniter more cuo axhibuisset, paulo post accidit, ut Gosselinus pest correptus interiret, et Diophanti codes eodem fato nobis eriperetur. Cum enim precibus meis motus Cardinalis amplissimus, nullisque sumptibus parcens, apud haeredes Gosselini codicem illum diligenter exquiri mandasset, et quovis pretio redimi, nusquam repertus est. (iii v)

80 The frontispiece bears the date of 1578, even though the dedicatory epistle is dated from november 1577.

In addition, there is the problem of the number of books of Diophantus. While only six survive, all manuscripts have as a title "The thirteen books of Diophantus." Du Perron's witness may not be completely reliable but it would be very surprising if Gosselin made a mistake about the number of books, just as he is announcing that he is devoting his talents to Diophantus. Certainly, we have to keep in mind that he made use of Xylander's version, for not only was the edition made in Basel and therefore accessible from Paris but Gosselin cites Xylander in the introduction to ***De Arte magna*** and calls him "the other Diophantus", but, most of all, Gosselin makes use of Xylander's terminology for aequations, such as *fictitia aequatio, duplex aequatio, adaequatio*, thereby showing a specific knowledge and appreciation of the text. So, we know that he worked for a long time on Xylander's version of Diophantus, but this does not mean that he would make no effort to ascertain whether the manuscript contained the last seven books. Quite the contrary. In conclusion, from Bachet, Du Perron, and Gosselin himself we have a witness of the existence of a complete manuscript of thirteen books. We can also conlude that, at least at the time of the ***De Ratione***, Gosselin was actually planning an edition of this text.

### *3. The publisher Gilles Beys*

We can learn more about the context of Gosselin's mathematical publication by examining what is known about his publishers. Let us start with the one we do not know, the publisher of the ***De Ratione***. This text is a unique gift copy, in parchment. It has not been possible to identify the publisher through the front matter such as letters or ornaments, in spite of a research as systematic as possible on Parisian publishers active in 1583, the date indicated in the frontispiece.[81]

The case of the other two publications is much more fruitful in this respect. Gilles Beys was a prominent *Imprimeur-Libraire* in Paris at the time.[82] Having started his activity by founding the *succursale plantinienne à Paris* in 1567, he marries one of Plantin's daughters in 1572 and publishes in Paris between 1577 and 1589, then in 1594 and 1595.

Beys does not appear to have a specific cultural project for publication. "Libraire besogneux, Gilles Beys n'est pas libre de choisir les livres qu'il édite et d'orienter sa production vers un domain particulier.[83]" However, from our specific perspective, some some publications provide interesting hints. First of all, we find, "imprimé à Anverse pour la succursale plantinienne à Paris", a text on double-entry bookkeeping by Savonne.

In 1577, after two literary works, Beys publishes Guillaume Gosselin's ***De Arte magna***. Then he publishes also the ***Historia imaginum caelestium*** by Jean Gosselin. This could indicate that Jean Gosselin is not necessarily the intermediary between Guillaume Gosselin and Beys, in spite of the difference in age and status.

Still in 1577, we find a work which we shall discuss in the fifth chapter, i.e. the ***Rhetorica*** by Omer Talon with the commentary by Claude Mignault. In fact, Mignault was *doyen de la faculté de droit*, and was a Ramist who showed, in his logical theory, an interest in mathematical thinking. Mignault was important in the history of the publisher, having suggested to Beys his printer's mark and motto. In his book on Alciati's ***Emblemi***, Mignault

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81 Thus far, the most likely candidate seems to be Jean Richer, who collaborated with Gilles Beys in previous editions.

82 See ***Imprimeurs et libraires parisiens du XVIe siècle***, tome III "Gilles Beys" (by Ursula Baurmeister) pp. 312-373.

83 Ibidem, p. 318.

writes: "ego illi (Gilles Beys) sertum albi lilii lubens exhibui addito boni Poeta hemisticho *Casta placent superis*." Another work by him will follow, the commentary to the ***Epistolarum libri duo*** by Horace.

In 1578, Beys publishes Cardano's ***De Subtilitate***. In 1578 we see Gosselin's translation of Tartaglia's work, accompanied by many dedicatory poems from the Académie de Baïf. In 1580 we notice two important works in the juridical field, the translation of Appian by Claude de Seyssel, Jean Robert's ***Animadversionum iuris civilis***, and Seneca's works. In 1581 appear get the poetic works of Jacques Courtin de Cissé we have already commented upon, as well as the continuation of the translation of Synesius of Cyrene. In 1583 Beys publishes ***Les Quatre premiers livres de l'univers***, a scientific poem by Miles de Norry, the author of an ***Arithmetique*** in 1574, who was associated with the Académie de Baïf.[84]

In 1583 Beys also publishes a work by Renaud de Beaune. In 1585 we find again an important juridical work, by Pierre Du Faur. Also Etienne Pasquier publishes his ***Poemata*** at Beys, in 1585 and 1586. Renaud de Beaune publishes his ***Psaumes de David*** in two editions, in 1587.

We can conclude that Beys represents well the cultural taste of the most educated élite of the period. In particular, Beys published several works by members or associates of the Académie de Baïf and the Académie d'Henri III.

### *4. Conclusion: Peletier and Gosselin*

Even though Gosselin's activity develops while Peletier is still alive, we see that the

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84 See Davis 1958.

two authors represent two different phases of the French algebraic tradition, as well as two different phases of Parisian high culture.

Peletier belongs to the previous generation, connected to the Court, thus to the first academies: the circle of Marguerite de Navarre and the Pléiade, as well as the first phase of the Académie de Baïf. He is active in his relationship to the process of publishing and with publishers, within the first movement for the French language in poetry and science.

Gosselin represents the time in which Tartaglia's arithmetic and algebra were so well accepted in the literate world that his work was published prefaced by poems. He was connected to the Académie de Baïf only in its second phase, when Latin began to be once again a crucial language, for many reasons. One was that those interested in the new cultural developments found positions at the Collège Royal, where instruction was in Latin. In particular, colleges were the realm of transmission of that theory of dialectic of the "Ramist" type, in Latin, to this and to the following generation.

Another reason for the increasing importance of Latin was the new ideal of the Orator. We have seen Peletier theorizing about the radical distinction between the Poet and the Orator, in which the Orator is seen as inferior, because the Poet is free to treat any topic.

In the second phase of the Academies, even Ronsard had to face a change. Now there was a new tendency to conceive of the Poet as "ignorant" and subject to a creative "fureur", incapable of debating rationally. In fact, we read in Yates that "the poets were displeased with the turn which the academies took under Pibrac and Henri III."[85]

This is the phase in which Davy du Perron says that "the science of words seeks either the truth which words hide, and is called Poetry, or the truth which words manifest,

85 See Yates, p. 128.

and is called Dialectic."  Not only was Davy du Perron a major figure in the Academies, but he was also connected to Peletier.[86]  Furthermore, as we have seen, Du Perron was from Caen, and knew Guillaume Gosselin so well that he gave him the most precious manuscript, the "thirteen books of Diophantus."  In the person of Davy du Perron, we have what seems to be way of establishing a clear connection between two very different people such as Peletier and Gosselin.

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86 See Yates, p.129.

# **Chapter II**

## **Arithmetic**

## **The abacist tradition between theory and practice**

***Part A. Arithmetic in France and Peletier's Contribution***

Early in the sixteenth century, arithmetic was the object of great interest by humanists and publishers in both of France's main centers, Lyon and Paris.

Taught at the university, arithmetic was in Latin. The curriculum included both speculative arithmetic, based on Nicomachus and Boethius, and practical arithmetic, i.e. the four operations, based on Sacrobosco's ***Algorismus***. Printed books followed this twofold tradition. The first important example of French arithmetic is the ***Arithmetica Severini Boetii in compendium redacta sive introductio in Arithmeticam speculativam Boetii, cum Jodoci Clichtovei commentario et astronomico libro Jacobi Fabri Stapulensis, quibusdam Caroli Bovilli lucubrationibus*** by Clichtove, Lefèvre d'Etaples and Bovelles, published in Paris by H Estienne in 1503, a book which was reprinted several times. In 1542 Oronce Fine, who had taught the first generation of French algebrists at the *Collège royal*, published his ***Arithmetica Practica*** (Paris, S. de Colines). This work, together with the entirety of his production, provides important evidence of a new interest in mathematics at the *Collège royal* and in the learned circles in Paris.

In addition, there were in existence works on astronomy: *Logistica*, which was for the Greeks simply practical arithmetic, was at this time the label for treatises on computation useful for astronomers. A late, but very significant, example of this is provided by Dasypodius' edition of Barlaam's commentary on the second book of Euclid and of Barlaam's ***Logistica astronomica***. Its importance is due to the fact that it was the first edition, and Gosselin considered it as a source for algebra. In fact, more space than is available here should be devoted to Euclidean arithmetic alone, for algebra was sometimes justified by the new (or the newly rediscovered) interpretations of Euclid. Until a more comprehensive study of sixteenth century French editions of Euclid can be made this will not be possible. But we shall then see that this Euclid had most of his impact in the second half of the century.

At the same time, there were the commercial arithmetics. In France and throughout the century, while these books were not treatises of bookkeeping they nonetheless gave instructions on how to solve many problems in commerce, such as division of profits, interest, money exchange, barter, and "alligation" (computations concerning the alloys used for money). As we have described in the introduction, these sorts of problems had been the object of study in abacus' schools and texts, although mostly outside of France. Yet, there are signs of interest in France as well. This is especially the case in Lyon, where we find two authors who did their apprenticeship in the abacus schools, Etienne de la Roche and Pierre Forcadel.

Etienne de la Roche is the author of a book in this tradition, ***L'Arithmetique*** (1520). It included some algebra, but was most of all a parallel to Pacioli's ***Summa***, collecting the main results of the abacus tradition. Amongst printed books, this example is the most famous and was a source for subsequent authors. It did not, however, stand totally alone. We should remember also two preceding works: ***Arismetique Corrige et Imprime a Paris*** (Paris, G. Nyverd, 1512) and ***Art et science de arismetique moult utille et proffitable a toutes gens*** (Lyon, 1515). They are anonymous, but this simply means they followed the common practice in the abacus tradition, in which texts mentioned at most the "paternity" of a single rule. Later French arithmetic manuals were "commercial" only insofar as they contained the rules mentioned earlier, but they also expressed an integration between learned and abacus school traditions. A telling instance of this new tendency is Forcadel's work.

But he develops the new genre that Peletier had started in France. To summarize the movement here, we see a period of two parallel genres, and then with Peletier's ***L'Arithmetique*** we see the integration of a learned and a "vulgar" tradition. While the content of this work has already been described,[87] we shall recall here some aspects of the rationale for Peletier's interest in arithmetic and algebra in particular.

As we have already seen (Chapter One above), Peletier theorized about writing science in French through an argument on *imitatio* and translation, and by way of a notion of the way a language and sciences are created and transmitted through a people. already in his ***L'Arithmetique*** we find most of Peletier's statements on these points, and we shall quote them at length later. But he had been and will be quite explicit also in other works. But, we might ask in particular, why focus precisely on the abacus tradition? An answer seems to come from Peletier's redefinition of the boundaries between theory and practice in the arts.

The relation between theory and practice is the topic discussed in the *proème* to the second book of ***L'Algèbre***, the one devoted to irrational numbers. Peletier writes:

> Ceus qui sont studieus des causes naturelles, Monseigneur, connoessent toutes choses ettre cpm[arties de deus Moetiez: lesqueles selon la differance de leurs Antiers, sont diversement nommees: Es Vivans, l'Ame e le Cors, es Rénes, le Conseilh e l'Execucion; es Ars, la Teorique e la Prattique, e an toutes les Sustances elemantees, la Forme e la Matiere. (...) E an chaque Tout, ces deus Parties sont telement affectees l'une a l'autre, e si

87 See Davis 1960 and Margolin. For our purpose it suffices to say that Peletier did not declare his sources for ***L'Arithmetique***, with the exception of Gemma Frisius, the work of whom he had already given an annotated edition in 1545. In fact, this work seems to be inspired most of all by Gemma. Cardano's ***Practica Arithmeticae*** had appeared in 1539, and could also be a source, but it is not cited. In conclusion, Peletier gives, like Gemma, an account of the algorisme and adds a set of commercial rules and problems. The novelties introduced by Gemma were in the choice of language (Latin) and the algorism, which was not presented in a practically oriented way, as in the original abacus style, but as a theory of numbers and their operations.

> mutuellement obligees: qu'on ne sauroet bonnement juger, laquele des deus e plus redevable a l'autre. E pour parler des Ars, comme etant ici notre principal propos: la Teorique e la Prattique sont deus seurs si gemelles, e ont une conspiracion si amiable ansamble: que l'absance de cete ci rand celle la sans profit, e l'absance de celle la, cete ci sans raison.

So, the two aspects of an art are so connected that their relation is compared to the Aristotelian synolon of form and matter. These two sisters agree so much that one looses its purposes without the other: theory will be useless and practice will be meaningless.

But this relation is reflected by relations between people:

> Le Praticien, avec son usance, bien souuent ne connet pas l'usage de leuvre: e si bien il antand que c'et, si ne set il quasi james e n'antand la raison de l'ouurage. E pour ce, a bon droet ét il reputè ignorant an son Art. Le Teoricien, sachant pourquoe il se fèt, e ne le sachant fere: peut justement etre estimè apprantis an sa Sciance: E tous deus ne meritet le nom que de demisavant.

Furthermore, writes Peletier, a person cannot reach perfection in both "halves" (actually, not even in half of one), and this keeps people in a "perpetual apprenticeship." The person who knows the most is the most aware of being ignorant, and becomes insatiable: "*vit an quelque delectacion, mais an continuelle pauvrete.*" By contrast, the ignorant is easy to please, "*vit heureus en son opinion*." In consideration of all of this, and taking into account that life is short, people devote themselves only to one of the two. Not only is it impossible to decide which one is the most important, but it is equally difficult to know for which one we are more talented.

Here Peletier's discussion of theory and practice comes to an end. It may well seem seem philosophically disappointing; especially after the triumphant beginning on Form and Matter. But in fact, Peletier contributes, with a skeptical and "sociological" argument, to an assessment of the question in which equal status is given to both theoreticians and practitioners.

Peletier's *proème* continues with a discussion on arithmetic:

> Je dirès qu'antre tous les Ars il n'y an point un, auquel l'homme puisse occuper sa cogitacion plus parfondemant, qu'an Aritmetique. E n'y a speculacion qui puisse seruir a l'homme de plus spacieuse campagne pour s'ebattre, pour entretenir ses pansees, pour se tirer hors de soe e puis se r'auoer, que l'universite des Nombres.

Thus far, on the usefulness of arithmetic. After praise for the infinite and for unity (*Vrey image de la Divinite*, later defined as *ce celeste Commancemant*), Peletier stresses the role of arithmetic in human knowledge of the world:

> Qui à il au Monde qui ne soèt signigiè voere conduit par Nombres? L'homme porte avec soè (s'il savoèt nombrer) le nombre de sa view, de sa fortune, de son gouvernement, de sa puissance e de son tout.

These are the philosophical statements made by Peletier in his ***L'Algebre***. They provide a good introduction to those we shall later find in Gosselin's texts.

In conclusion, Peletier was motivated by his program on French mathematics, but he wanted to stress the importance of the practical tradition, in particular of the abacus tradition which , with Gemma Frisius and Peletier himself, becomes the most important part of practical arithmetic. It is only fitting that this becomes explicit in the writing of his French algebra.

But arithmetica and algebra have a special role, as Peletier explains in the *proème* to the fourth book of ***L'Arithmetique***. The question is why we do not have Greek texts on practical arithmetic and on the use of arithmetic. Here Peletier tacitly assumes that the ancients had some rudiments of abacus arithmetic, and not simply what they called practical arithmetic, i.e. the four operations. There are two hypotheses: one is that the ancient authors considered practical arithmetic as typical of "*gens mecaniques*." The second is that they were actually so concerned by the theory that they could ignore the practice. The latter cannot be true, writes Peletier, because they had to **do** mathematics in order to know its theory. This is especially clear because they did not learn it, as we do, in a methodical manner.

> Les grans personnages du temps passé, qui avoient toute leur entente aux nouvelles et suttiles invention, se sont contentez de l'Aritmetique speculative, comme vrai et propre obget de l'esprit, se proposans qu'il ne doir chaloir a un Matematicien (lequel doir abstraire ses imaginations des choses maniables et corporelles) de se meller de regler les negoces et entremises des hommes.

So far, Peletier makes clear that the ancient authors made a choice in view of the Aristotelian idea of mathematics, i.e. as the science whose objects are obtained by abstraction from physical objects. But then he makes a new, less "theoretical" point: the case of arithmetic is the same as that of music, which also had gone through the same process:

> Nous voions mesme en la Musique, qui est un art de tous le loins actif, que les anciens ne se sont point adonnez a mettre rien en chant que sus les Instrumens, et non par ecrit, fors long temps apres l'Eglise Romaine introduitte. Et encores ce qui fut premierement redigé, etoit simple, nu, et quasi sans artifice: Car quant a la composition que nous appellons des choses faites, il n'est point de memoire qu'avant soissante an enca.

Evoking the case of music is, for Peletier, natural in many way: this was a mathematical discipline and a topic of great interest for the platonic circle to which Peletier belonged. Furthermore, it was a discipline which had become "literate" in Italy, after a long "vulgar" tradition comparable to the abacus tradition in mathematics. For, we should remember an important fact: Peletier knew Italian, while he did not know German. Furthermore, Italian humanism was at the origin of many aspects of the new French humanistic movement in which Peletier was involved. While the names of Italian mathematicians appear only in his later text, ***L'Algèbre***, Peletier was aware of "translating" into French culture the transformations from practical into literate knowledge which had been typical of recent Italian culture. But on this point, let us see the other French algebrist, Guillaume Gosselin, at work on an actual translation.

### *Part B: Gosselin's translation of the General Trattato*

#### *1. The context*

In March 1578 the young mathematician Guillaume Gosselin, resident at the Collège de Cambrai of the University of Paris, published a translation and abridgement of the ***General Trattato***[88] of Nicolò Tartaglia.

Published twelve years after the first Venetian edition, this translation[89] was the first ever made of the *General Trattato*. The interest in Tartaglia's work seems to have continued, since in 1613 another publisher produced a reprint of Gosselin's text.[90]

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88 The work is divided into two parts, each of them having its own frontispiece. The first part bears the title: "La prima parte del General Trattato di Nicolò Tartaglia, nella quale in diecisetti libri si dichiara tutti gli atti operativi, pratiche et regole necessarie non solamente in tutta l'arte negotiaria, & mercantile, ma anchor in ogni altra arte, scientia, over disciplina, dove intervenghi il calculo", in Vinegia, per C. Troiano dei Navò, 1556. The second part bears the title "La Seconda parte del General trattato di numeri e misure di Nicolò Tartaglia, nella quale in undici libri si notifica la più elevata, et speculativa parte della practica Arithmetica, laqual è tutte le regole, et operationi practicali delle progressioni, radici, proportioni et quantità irrationali", by the same publisher.

89 The work bears the following title: "L'Arithmetique de Nicolas Tartaglia Brescian, grand mathematicien, et prince des praticiens. Divisée en deux parties. Recueillie, & traduite d'Italien en François, par Guillaume Gosselin de Caen. Avec toutes les demonstrations Mathematiques: et plusieurs inventions dudit Gosselin, eparses chacune en son lieu. A Paris, chez Gilles Beys, rue S.Iacques, au Lis blanc. 1578, avec privilege du roy."

90 Cfr. Adrian Périer, 1613. We should notice, though, that this publisher was the heir of the previous publisher Gilles Beys (cfr. Philippe Renouard, ***Imprimeurs et Libraires parisiens au XVIe siècle***, vol. III, p.312, "Gilles Beys." Kästner devoted to this text a few pages of his ***Geschichte der***

Why did Tartaglia's work, and later Gosselin's version, attract attention in France? Does the diffusion of the ***General Trattato*** and of its reduction point to an affinity in the origin and the public of the work in the two countries, or suggest that both the context of production and of use of Tartaglia's book beyond the Alps, was similar to that in Italy? Did the use of Tartaglia's work, as proposed by Gosselin correspond to its author's intentions? We cannot give an exhaustive answer in the context of these pages. Rather, we shall limit ourselves to examining in the two texts the question of the introduction of the abacus tradition (commercial arithmetic and algebra) into the quadrivium, and more precisely the relationship between theoretical and practical arithmetic. In this way, we may get a fuller picture not only of the milieu to which these texts were addressed, but also the way in which the authors themselves conceived of their works.

***2. The mathematical thought of Tartaglia***

Tartaglia begins his work with a dedicatory letter of philosophical character, in which he addresses the fundamental question of the relationship between theory and practice. He writes:

> Gli antichi sapienti, honorando signor compare, (come scrive Ptolomeo nel principio del Almagesto) dividerno la sapientia in due parti, la prima delle quali dal detto Ptolomeo è detta speculatione e l'altra è chiamata operatione. (...) Il fine della scientia speculativa, (come dice Aristotile nel secondo della Metafisica) non è altro, che la verita della operatione, over pratica dell'opera compita, & abenche la speculatione (per esser investigatrice delle propinque cause, et augmentatrice della scientia) sia molto piu nobile della operatione, over pratica operativa, la quale solamente attende a sapere con diligenza esseguire, & condur attualmente a fine, over ad effetto tutte le cose gia speculativamente ritrovate, notificate, et regolatamente in atto poste, nondimeno per quanto posso considerare, a me mi pare, che quanto piu la parte speculativa ecceda di nobiltà la parte operativa, tanto più la parte

***Mathematik***, Göttingen 1796, I p.197-200.

> operativa ecceda, non solamente di utilita, la parte speculativa, ma anchora di laude, perché, come dice M. Tullio nel primo de officis, ogni laude della virtù consiste nell'attione, over operatione.[91]

This is taken from the dedicatory letter of the first book, but it refers to the whole work. The theme of practice is in fact very important and occurs often through the ***General Trattato***, with, however, a special role in this first part, which it is devoted to the calculus of the four operations with particular stress on the calculus useful in a commercial context.[92] Here Tartaglia's thesis is that, while speculative mathematics has a higher status, practical mathematics is more praiseworthy. Tartaglia has already translated Euclid, thereby contributing to speculative mathematics. The ***Trattato*** must now provide the tools to make use of that mathematics, i.e. to **operate** in arithmetic and geometry.[93]

Tartaglia's sources are classical, but he has recourse to *topoi* which are not often

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91 Given the rarity of this work, as of Gosselin's, I have chosen to quote extensively.

92 See the subtitle of this second part in note 1. These are the operations on integers and rational numbers and the rules for the change, the alloys, the barters, the interests, the inheritances, which derive from the rule of three and the rules of false position. This is however not what we would call algebra. Much space is reserved for the relative commercial problems, which exemplify the artifices necessary for applications. The mathematical content of the Second Part is clear from the title. Gosselin does not deviate from Tartaglia's text in the choice of mathematical content.

93 So writes Tartaglia later in the dedicatory epistle. The outline of the work announced in the preface and the posthumous edition of the ***General Trattato*** consists of six parts. The third and the fourth part were printed before Tartaglia's death, in December 1557, as is apparent from the inventory of his goods (see Giovanni Battista Gabrieli ***Nicolò Tartaglia, invenzioni, disfide e sfortune,*** Quaderni del centro studi della Matematica Medioevale, Università degli Studi di Siena, 1986), though they bear, like the last parts, the date of 1560. The Third part is devoted to mensuration and to practical geometry, the Fourth Part to speculative geometry. The Fifth and the Sixth Part were actually printed after the author's death, and we do not even know to what extent they coincide with Tartaglia's own text. In any case, the Fifth Part is devoted to the solution of problems by means of ruler and compass, whereas the Sixth Part is devoted to algebra. At the Bibliothèque Nationale in Paris and in other French libraries there are preserved various copies of the 1560 edition, in six parts.

found in the introductions to mathematical books. For we know to what extent, in late humanism, the age of quotations and of books of topoi, it is necessary to make the distinction between various sorts of *topoi*. The diversity of Tartaglia's choices is evident not only from the authors mentioned (Aristotle, Isidore, Boethius and Sacrobosco, but also Michele Scoto and Girolamo Savonarola) but even more from the passages quoted from these authors and the use made of them.

As we have noted, the theme of practice occurs often in the ***General Trattato***. It appears, first of all, in the reinterpretation of the Aristotelian distinction between the physical point of view and the mathematical point of view. In fact, Tartaglia often mentions this Aristotelian distinction which comes from the second book of the ***Physics***:

> We must now observe in what respect the mathematician is different from the physicist. The mathematician does not study the attributes insofar as they are attributes of such beings. In this way, he also makes a separation: in fact, for thought, they result separable from motion, and there is nothing wrong if this happens, nor do they fall into error those who operate such a separation. [193b]

Aristotle also mentions this distinction often in the ***Metaphysics***, where he deals with the classification of sciences. Tartaglia writes:

> Bisogna notar qualmente vi sono de due sorti considerationi, l'una è del Naturale, e laltra è del Mathematico, il Naturale considera le cose secondo lesser come secondo la ragione congionte con qualche materia sensibile. (G.T.p.2v)

This is, according to Tartaglia, what Aristotle and Savonarola maintain, and he refers explicitly to the sixth book of the ***Metaphysics***, and probably to the famous passage:

For, physics deals with beings which exist separately, but are not without motion, and in turn mathematics deals with beings which are in fact unmovable, but which maybe do not exist separately and are as present in a matter; instead science deals first with things which exist separately and are unmovable. [1026a]

Already the notion of unity is an instance of this thesis, because, as Tartaglia writes,

...il detto Naturale (...) sempre la nomina congiuntamente insieme con quella Materia sensibile, cioe con quel suo material sugetto, digando un ducato doro, over un scudo, over un fiorino, over una lira, over un soldo, over un danaro, over un brazzo di panno, over una lira de seta, over una marca doro, over una onza de zafrano, over un caratto di muschio, & similmente nelle misure geometriche, digando una Pertica, un Passo, un piede, una onza, & così nelle misure di astronomia digando un grado, un minuto, un secondo, e così nelle parti, digando un mezzo brazzo di panno, un terzo de un ducato, el quarto de onza de oro, & così discorrendo in tutte le altre cose materiale, che occorre nell'arte negotiaria, over mercantile, & altre. Et queste tali specie de unità convenienti se possono chiamare unita naturale, over denominate, & queste tale sono divisibile in infinito in quanto alla quantità di quel suo materiale soggetto.(G.T.p.2)

Tartaglia then describes the point of view of the mathematician:

Il Mathematico poi considera le cose per congiunte secondo lessere, con tal materia sensibile (si come fa anchora il Naturale). Ma le piglia, over considera poi come astratte da tal materia sensibile secondo la ragione.

Finally Tartaglia devotes a whole section to the "*comparatione della consideratione del Natural, e del Mathematico sopra la unita, e della differentia di quelle*":

Acciò che meglio se apprenda, over intenda da ogni qualità di persone, la differentia di queste due sorti de considerationi, cioè del Naturale, e del Mathematico, sopra la unità, e la differentia delle dette unità, cioè Naturale & Mathematica, pongo questo caso, che siano huomini, che considerino uno medesimo Animale, et laltro consideri solamente l'anima del detto animale, hordico che la consideration del primo, è simile alla consideration del naturale, e quella del secondo è simile alla consideration del Mathematico. Et perché il corpo di tal animale è una materia sensibile, & divisibile secondo la quantità, diremo quel tal corpo esser simile alla unità Naturale, similmente perché l'anima del detto Animale, è una cosa insensibile, & indivisibile diremo quella esser simile alla unità Mathematica, la quale unità Mathematica Carlo Bovile per molte sue ragioni dice ch'ella è da esser comparata al Summo Iddio, & per questa come tengo, che li nostri antichi savi attribuirno questo nome de unità al detto nostro summo Architettore. (G.T.p.2v)

Of interest is this interpretation by Tartaglia of the theory of mathematical beings as obtained *ex aphaireseos*, by abstraction, and which gives foundation, in Aristotle as in Tartaglia, to the speculative status of mathematical science.[94] Further, it should be noticed that Tartaglia includes a reference to Charles de Bovelles,[95] which would most likely have been known to Gosselin, given the importance, in France, of Bovelles' edition of Boethius' ***Arithmetica***.

Tartaglia stresses the contrast between the points of view of the mathematician and of the natural philosopher in the following chapter, devoted to the notion of number:

Il Numero (come diffinisce Euclide nella seconda diffinizione del settimo)

94 And it guarantees at the same time separability from bodies and non-independence from them. The redefinition of mathematical sciences in this period goes inevitably through this theory, as much in the Aristotelian versions as in the one given by Proclus, but space limitations do not allow us to develop this point.

95 Author of a practical geometry, and co-author of the ***Praxis numerandi*** of 1503, together with his master Jacques Lefèvre d'Etaples and Josse Clichtove. It is a treatise of practical arithmetic in the classical sense, as will be explained below. On the role of these three authors in the reevaluation of the mathematical studies among French humanists of the XVIth century, see J.C. Margolin ***L'enseignement des mathématiques en France (1540-1576)***, in Sharratt (ed.) "French Renaissance Studies", Edinburgh 1976, pp. 109-155.

non è altro, che una Moltitudine composta delle unitade. Ma bisogna avertire, che sopra el numero vi son quelle medesime due sorte di considerationi, dette sopra della unità, cioè una secondo il Naturale, & l'altra secondo il Matematico. Il naturale considera il detto numero, si secondo la ragione, come secondo lessere, congionto con quelle materie sensibili numerate, cioè con quel material soggetto, di quelle unità naturali, componente quel tal numero, e però sempre proferisse, et denomina il detto numero, congiontamente insieme con il detto material soggetto, digando, over tanti ducati, over tanti scudi d'oro, over tanti fiorini, over tante lire, over onze di zucaro, over di canella, over di zenzero, over altre materie simile, over tante Marche, once, quarti, over caratti di oro, over argento, over tanti staia, quarte, over quartaroli di formento, over altro grano, & così discorrendo in tutte le materie occorrenti nelle monete, pesi, & misure, si geometrice, come non geometrice (come fu detto della unità Naturale) e pero questi tai sorte di numeri si possono convenientemente chiamar numeri naturali, over denominati. Ma il Mathematico poi considera il detto numero, si come una moltitudine composta de unitade Mathematice, cioè astratte da ogni materia sensibile secondo la ragione, cioè indivisibile secondo la quantita, & tal specie de numero convenientemente se gli puo dir numero Mathematico, e questo medesimamente afferma Aristotele.

It is easy to see the shift in meaning between Aristotle's and Tartaglia's distinctions. For Aristotle, it is necessary to acknowledge the difference between the two sciences in order to assure the distinction of genera which guaranties the rigor of demonstrations. If Aristotle's polemical target is the Platonism of mathematical beings, the outcome of his theses is a certain methodological Platonism which would create the conditions for a contemplative science of abstractions by defining its ontology.

According to Tartaglia, the point of departure is to draw a line of demarcation between mathematics as applied to concrete problems and mathematics which studies mathematical beings and related problems, which can be useful for solving entire classes of practical problems. The traditional partition did not convince him, as we can see from the contents and structure of the ***General Trattato***. In fact, each chapter of the ***General Trattato*** includes both a part that we would define as theoretical (and which Tartaglia sometimes calls speculative), intended to solve classical Greek problems, Euclidean or not, and a part on applications, for instance commercial ones. The same alternation is recognizable between whole parts. This was not usual in the works of the time. For instance, the ***Practica Arithmeticae*** by Cardano (Milano 1539), though introducing many innovations into the genre of algebraic treatises, did not give to rules that general formulation which is characteristic of the ***Ars magna***, which came soon afterwards. Cardano's choice depended therefore on the fact that his text belonged to the practical genre.

But the order of the ***General Trattato*** modifies the traditional one in a more radical sense, thanks to the inclusion and the reciprocal articulation of themes of theoretical mathematics and of practical mathematics, which up this time had always appeared in separate texts.

Let us now return to the initial theses articulated by Tartaglia on the distinction between theoretical and practical arithmetic:

> Le specie della Arithmetica sono due, cioe Theorica, & Pratica. La Theorica considera le cause, le Qualita, le Quantita, & le Proportion de Numeri con una Speculation di mente, & il suo fine, non è altro che la verita, & di questo abondantemente ne tratta il nostro precettore Euclide Megarense nel suo Settimo, Ottavo e Nono Libro delli quali al suo loco, & tempo in pratica ne parleremo.

It should be noted that Tartaglia does not mention the second book of the ***Elements***, which in this period was reinterpreted in algebraic terms, nor the tenth, which was cited by Tartaglia himself and by Stifel in the treatment of the irrational numbers. Instead, he mentions those books which belong to a tradition of speculative arithmetic, from Nicomachus and Boethius up to Bovelles. Tartaglia goes on:

> La Pratica poi, considera solamente l'attione, over Calculatione, & il fin suo non è altro, che il compimento di tal attione, over calculatione, e di questa pratica è lo intento nostro di voler abondantemente trattare, incominciando prima dalle prime attioni, pratiche, & Regole generali, & particolari pertinenti in tutta larte Negociaria, over Mercantile.(G.T.p.1v)

Here "attioni" means "operazioni." At first sight the disciplinary partition proposed by Tartaglia, in its most general form defining the two parts of arithmetic, seems to follow tradition, dividing the nature and classification of numbers (*speculative arithmetic*) from the operations performed on them (practical arithmetic).

It should be noted that the text of the ***General Trattato*** resembled to some extent in both its problems and its style some university texts, like the ***Arithmetica integra*** of Stifel (1543).[96] At least as far as genre is concerned, Tartaglia's is the sort of work that might have been used in universities, or rather by students outside of their formal studies.

Besides making a fusion of the two fundamental types of arithmetic, the ***General Trattato*** took its arithmetic from the abacus tradition, and assumed most of its contents (commercial problems) and the internal organization proper to it just as Stifel had done, but relying more heavily on the Italian tradition. Furthermore, a comparison of the ***General Trattato*** with Pacioli's work shows that Tartaglia added to the contents of the ***Summa de aritmetica*** not only sixteenth-century developments in algebra, but also the experience he acquired in translating Euclid. As with Stifel, this put into question the very notion of arithmetic. The attribute of "integra" represented the inclusion of operations on irrational numbers and on cossic numbers, i.e. of algebra, and this at a level of generality which, for sixteenth-century authors, was possible with recourse to the tenth book of Euclid.

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96 As an anonymous reader of the XVII century wrote on the frontispiece of the copy of the ***General Trattato*** now at the Biblioteca Ambrosiana, the text by Tartaglia is "Michele Stifelio sopra Euclide applicato."

Even the conception of the ***General Trattato*** is therefore theoretically represented by Tartaglia's discussion of the Aristotelian distinction of the point of view of the natural philosopher and of the mathematician. The tradition of commercial arithmetic (as well as that of practical geometry[97]) takes its rightful place in mathematics, and it would be a mistake to identify it with the point of view of the natural In it one must distinguish the parts that take such a point of view, such as the countless practical problems (evoked by Tartaglia as examples of the concept of unity and number) and their solutions by means of artifices, and on the other hand, those that adopt the mathematical point of view, such as the calculus of cossic numbers and the theory of equations. The "natural" part is based on the concept of *arithmòs*, which was always a number of things,[98] while the "mathematical" part introduces the cossic number or letter, which is the number of number, the *intentio secunda*, and which furthermore operates with the general notion of number: this includes the irrational, the negative and even (sometimes), the imaginary: in a word, the algebraic number.

Hence, theoretical and practical arithmetic are no longer distinguished according to their object, but according to their function and goal. The discussion of a particular object, for instance the irrationals, leads the author to treat them both from the point of view of the natural philosopher and from the point of view of the mathematician.

This is not in contradiction, but instead in accordance with the reevaluation of

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97 We do not take into consideration this aspect, though important, of the ***General Trattato*** because it is not taken up by Gosselin. For this topic in general, see the classic article by Natalie Davis, "Sixteenth-Century French Arithmetics on the Business Life." ***Journal of the History of Ideas*** Vol. XXI, No. 1 1960, because it deals with the transformation of all the practical mathematics in the second half of the XVIth century.

98 This distinction and its transformations in the sixteenth century constitute the object of the fundamental and well known text by Jacob Klein, ***Greek Mathematical Thought and the Origins of Algebra***. Cambridge Massachusetts. M.I.T. Press, 1966.

practice. Such a reevaluation is necessary first of all to allow the abacus tradition to emerge, to be taken into consideration at the level of Euclid. Euclid developed the theory, the abacus tradition developed the practice, and both traditions are "worthy of praise." Secondly, such a reevaluation allows us to recognize that a speculative part exists within the practical tradition. The conclusion is that theoretical and practical arithmetic are defined reciprocally as articulations of the same discipline, and do not constitute two radically distinct fields of knowledge.

To be sure, Tartaglia's interpretation is only one of the many given, in the sixteenth century, of the integration of abacus arithmetic into university teaching or at least into the learned literature. For example, Commandino does not hesitate to situate commercial arithmetic and algebra in the context of practical arithmetic, because he defines as practical the arithmetic which calculates the problems of commercial life, and as theoretical the arithmetic that "uses imagination *tamquam abaco*"[99] thus updating, in the most rigorous sense, the disciplinary and social distinction introduced by Plato between Arithmetic and Logistic.[100]

In fact, Commandino and Tartaglia were at cross-purposes on this point, in spite of their reciprocal admiration.[101] Tartaglia meant to propose a text which corresponded to the

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99 See the essay by E. I. Rambaldi, "John Dee and Federico Commandino: an English and an Italian Interpretation of Euclid during the Renaissance", ***Rivista di Storia della Filosofia*** n.2, 1989.

100 See, also in this connection, the already mentioned book by J. Klein, especially the chapters 2 and 3.

101 In particular, the dedicatory letter (to conte Antonio l'Andriano) of the second part of the ***General Trattato*** includes a reference to Commandino: "Et perche gia molti giorni ragionando con la eccellenza di messer Federico Commandino da Urbin peritissimo mathematico, quella mi certificò qualmente vostra Signoria molto si dilettava, non solamente della speculativa dottrina di Euclide Megarense, ma anchora della pratica speculativa dell'arte magna." See also the letter relative to the debate between Tartaglia and Commandino about the cubic roots, in P. L. Rose ***Letters***

new social "status" of abacus mathematicians.[102] They were in fact asked, like Tartaglia, to decide practical questions of various kinds -- classical commercial questions, but also ones of inheritance, surveying and so on.[103] The ***General Trattato*** is therefore first of all the broadest summa of the abacus tradition after that of Fibonacci, but Tartaglia adds on the one hand, practical arithmetic and on the other, classical geometry. These various elements, integrated into a unified plan, constitute a new image of mathematics, which stresses the role of "practice" in such a way as to show its universality, in philosophy, in theory and in applications. The classical character of the discipline is demonstrated by trying to "show off" a wide philosophical culture, citing authors and questions which belonged to the speculative tradition. It is plausible to maintain that, on the contrary, Commandino, like Baldi and other aristocrats from Urbino, needed new categories which could "ennoble" their practical mathematics (mechanics, fortifications, military art), thus setting it apart from the commercial applications of shops and abacus schools.[104]

***illustrating the career of Federico Commandino (1509-1575)*** "Physis" 1973.

102 I agree, on this point, with the theses presented recently by M.Biagioli in "The social status of italian mathematicians, 1450-1600", ***History of Science*** no.75, March 1989, especially pp.59-61. See also the classic P. L. Rose ***The Italian Renaissance of Mathematics*** Genève, 1975 and now, E. Gamba, V. Montebelli ***Le scienze a Urbino nel tardo rinascimento***, Urbino 1988. The latter contains also a discussion of the philosophy of mathematics presented by Tartaglia in his translation of Euclid.

103 See for instance the recent biography of Tartaglia, already cited, by G. B. Gabrieli, 1986, p.21 and ff.

104 See in this connection the already cited essay by Biagioli.

### *3. Gosselin's mathematical thought*

Reflection on the role of practical mathematics is entirely omitted in Gosselin's version, and he also reduced the introductory pages which deal with philosophy of mathematics -- definitions of arithmetic, unity, and so on. This is all the more meaningful because, in other respects, Gosselin gives a lot of space to reflection on mathematics, which is unusual in his version of the ***General Trattato*** as a whole.[105] It is therefore clear that Gosselin is aware of the need to cut these parts, and the reasons for his choice are various. First of all, he does not share either the overblown style of Tartaglia, typical of an autodidact, nor the theses of his philosophical discourses, nor finally and more fundamentally the cultural project that corresponds to it. For Tartaglia's treatise, rich in demonstrations, written in classical style, and oriented to practice, Gosselin substitutes a manual. It is certainly thicker than previous French manuals of arithmetic, but certainly slimmer than Tartaglia's original. Here the orientation is didactic. The point is to acquire some simple techniques, which might be developed elsewhere, beyond the official teaching, and to integrate them into a style of thought, in the tradition of Peletier's ***Arithmétique*** (Poitiers, 1548), a commercial arithmetic for an aristocratic culture. The use of such techniques in practice is not at all the primary target. Rather, the author wants to introduce the otherwise cultivated reader to this type of mathematics.

On the other hand, Gosselin also formulated a philosophy of mathematics, particularly developed in his work ***De ratione discendae docendaeque mathematices***. In it we find among other things a distinction between theory and practice through which he expresses simultaneously his affinities and differences with Tartaglia. At first Gosselin

105 In fact, Gosselin's version is abridged overall, with respect to the original, but nowhere else does he omit an entire section or aspect of Tartaglia's thought.

writes:

> Iterum utraque quantitatis species bipartito dividitur, in agentem et cognoscentem: haec leges facit, regulasque condit, illa cognoscenti innixa quam volebat actionem consequitur; prioris sunt Problemata, Theoremata posterioris.(p.7)

This distinction holds for the whole of mathematics. It is later clarified in connection with the one between geometrical theorems and problems, a typical *topos* of the time, because of its importance for Proclus, who had been recently rediscoved:

> Propositionum duas esse species diximus, Theorema et Problema, quarum prior contemplatur, posterior in opere est: neutra vero sine altera consistit, utraque ex finitionibus, principiis et petitionibus nascitur.(p.11v)

We shall return to this text later. What matters here is that Gosselin, like Tartaglia, saw the mathematical advantages of Stifel's approach, which integrated the abacus tradition of commercial arithmetics and algebra with Greek mathematics by means of an updated reading of Euclid. The philosophical justification for this approach is different from that of Tartaglia. Similarly, the space devoted to the abacus tradition proper, together with the series of problems and tables of change which characterize them, is very much reduced. Gosselin neglects many references to geometry and develops those aspects of the abacus tradition which Tartaglia defined as "speculative."[106] The transcription in Euclidean terms is no longer necessary for Gosselin, first of all because he takes as given, at least in part, that it can be done. Furthermore, the richer parts of the abacus tradition, such as the extraction of roots, the study of irrational numbers, and algebra, have now acquired their cultural dignity

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106 I refer here to the Second Part of the ***General Trattato***, the subtitle of which is given in note 1.

within mathematics.

Let us examine more closely how Gosselin establishes the inclusion of algebra within the liberal arts.

Gosselin gives algebra the highest position among the mathematical disciplines and justifies this by stating that it is the most general among them. In fact, in his ***De arte magna***,[107] Gosselin attributes to algebra, not to arithmetic, the theory of *exemplaria*, using the same words that Tartaglia had reserved for numbers:

> Huius scientiae quae ab antiquis appellata est scientia creaturae et creaturarum, ab aliis regula regularum, ab aliis denique regina scientiarum, tota ratio in proportione occupata est.(***De Arte Magna*** p.3)

Tartaglia had, in fact written:

> E pero tutte quelle cose che dalla primeva origine hanno avuto producimento per ragion di numeri sono state formate, e così come sono debbono essere conosciute, come dice Boetio, et Giovanni di Sacrobusto (...). Questo fo el principal essemplare nel animo del conditore. Da qui ne è nasciuta la moltitudine di quattro Elementi. Da qui ne nascono i movimenti delle stelle, e le conversioni di Cieli, da qui tutte le cose create si reggono sotto ordine de numeri, E pero nella cognitione di tutte le cose questa Scientia, over disciplina, e necessaria, neanche cosa al mondo se trova, che senza numero questa Scientia, possa stare. Egli è adunque la Arithmetica, scientia de numeri over (secondo alcuni) scientia del Creatore, & delle Creature, la qual sotto coprimento de numeri dimostra la sua cognitione.(G.T.p.1v)

The function of this *topos* in Gosselin's text is similar to the one made explicit by Tartaglia in another passage devoted to the theory of *exemplaria*. Here we understand that

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107 ***De arte magna, seu de occulta parte numerorum, quae algebra et almucabala vulgo dicitur***, Parisiis, apud Aegidium Beys, 1577.

Tartaglia's emphasis on the role of numbers in the constitution of the universe is his way of adducing a theological argument in favour of the dignity of arithmetic and of its use in the world. Already in the second section of the first book, "What is arithmetic" Tartaglia summarizes the Aristotelian doctrine of abstraction and proposes a theory of arithmetical *exemplaria* for the creation. He writes:

> L'Arithmetica adunque... è Disciplina de quantita discreta, cioè numerabile secondo se, chiamata da alcuni vertute de numero per esser tutte le cose alla sua similitudine formate.(...) Lo approvaremo per Severin Boetio, qual nel prohemio della sua Arithmetica così dicendo scrive, Quale adunque di queste Arti Scientie over discipline liberali è quella la qual prima si debba imparare se non quella la quale come principio, & matre ottiene alle altre la portione. Questa certo è l'Arithmetica. Questa veramente è de tutte la prima. Non solamente perche , quel summo di questa Mondial Machina Conditor Iddio prima hebbe essa per un esemplare della sua ratiocinatione inanti agli occhii, A questa tutte le cose le quali lui ordino sono concordate fabricate la ragione per li numeri dello detto ordine.(G.T.p.1)

The theme of the *exemplaria* was present in various ways in sixteenth-century mathematics, mostly in the idea of Adamic mathematics, the fundamental kernel of human knowledge. According to this point of view, arithmetic could be the basis for **action** on nature, not only of contemplation of the truth.[108]

If Gosselin therefore takes up a theme already elaborated by Tartaglia, it is also true that he applies it to algebra, to which he granted a preeminent role in relation to the rest of arithmetic. Of course, this implies a transformation of the contents. Tartaglia had limited himself to giving value to some aspects of the algebraic tradition, by incorporating them into

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108 In this sense we can recall the theses expressed by John Dee, treated by the article by Rambaldi already cited. Of particular interest would be a comparison between Dee's position on the three modes of number (in the Creator, in each Creature, in the minds) with the Numero Numerans, numeratus e numerabilis recalled by Tartaglia with explicit reference to Albertus Magnus, Michael Scotus and Pier Lombardo.(G.T.p.3)

speculative arithmetic. Gosselin does not hesitate to call *algebra* that part of it which constitutes its theory, the general part: definitions, calculus of algebraic numbers, theory of equations.

***4. Algebra***

Gosselin brought us directly to the subject of algebra. Tartaglia's work, instead, deals with algebra only at particular points in the second part, and obviously in the posthumous sixth part. Great importance is however attributed to algebra, as we can see already in the dedicatory letter:

> Deliberai nella mente mia di componere a comun beneficio un general trattato di numeri & misure, si secondo la consideration naturale, come Mathematica, & non solamente nella practica Arithmetica, & di Geometria, & delle proportioni, & proportionalità, si irrationali, come naturali. Ma anchor nella pratica speculativa dell'Arte Magna detta in Arabo Algebra & Almucabala, over regola della cosa.

In fact, only the sixth part of the ***General Trattato*** would be devoted to algebra. It constitutes a very updated treatise in this discipline, written in an elevated vernacular style, and containing geometrical demonstrations of algebraic formulas. There is a reasonable possibility that the text approaches Tartaglia's actual writings, of which some manuscripts apparently circulated. This fact was known also to Pedro Nunez, as he explains in one of the most important algebra manuals of the time.[109] What concerns us here is that Gosselin, who limited himself to abridging the two first parts of the treatise, considers the Second part the one that establishes the foundations of algebra. We have seen that this was also Tartaglia's

109 ***Libro de Algebra in Arithmetica y geometria*** Anvers 1564.

intention, since he cites in this connection a conversation with Commandino. Gosselin writes to Marguerite de France, in the epistle, that the science of number is made up of three parts:

> ...la première desquelles a retenu iusques à present le nom du genre, & est appellée Arithmetique, de laquelle nostre Autheur a amplement discouru en la precedente partie; la seconde est dite Musique, laquelle desia se recule du nom de science, pour estre dependante des accors & concordances des sons, lesquels sont maintenant plus fors, maintenant plus foibles, eu égard à la Matiere; la troisieme est appellée d'un mot Arabic, ou Algebre, ou Almucabale, laquelle est toute fondée sur les proportions, plus secrete, subtile, & divine qu'aucune des autres parties: pour parvenir à laquelle, nostre Autheur enseigne les principes, fondements, supputations d'icelle, en ceste seconde partie, attendant qu'en la sixième il en baille l'usage & equation: de laquelle partie ce grand Arithmeticien Eudoxe a tiré et premier inventé les hypotheses de vostre Astrologie, par le moyen de ceste proportion continue, laquelle elle garde.(***L'Arithmetique***, seconde partie, p.iii)

In fact, in the Second Part of the ***General Trattato***, only one chapter, the second, concerns what we consider algebra. In it, Tartaglia deals in a unified way with roots, both those obtained by extraction, and the unknowns, or roots of equations. First he deals with them in descriptive terms, in a paragraph bearing the title "Whence this name of root is derived." He writes:

> Si come che nelle erbe, e nelle altre piante, dalla natura prodotte, questo nome radice significa quella sua più bassa e originale parte occultata dalla terra, dalla qual tal erba over pianta è stata produtta, e generata, il medesimo, per similitudine, ogni numero vien detto Radice di qualsivoglia numero da lui medesimo produtto e generato, essempi gratia ogni numero dutto in se medesimo viene a esser radice di quel suo produtto, cioè 1 dutto in se medesimo fa pur 1 e cosi 1 producente vien a esser radice di quel suo produtto 1. (...). Ma si come, che dalla radice di una herba, over di una pianta si produce piu qualita di materie, cioe prima produce una certa piccola

> cosa appena apparente sopra a terra, da poi produce un fusto, over foglie secondo la qualita di tal radice, e dapoi produce fiori, e dapoi frutti overo semenze, onde di ciascuna di tai materie la detta prima radice, vien a esser sua radice, perche il tutto è stato produtto da tal prima radice, e dalle cose produtte da quella, cosi medesimamente intervien nel numero, perche ogni numero dutto, overo multiplicato in se produce il suo quadrato (detto censo) e tal numero vien a esser la radice di quel tal numero, e tal radice è detta radice quadrata, overo censa di quel tal quadrato.

In this way, Tartaglia introduces the powers of the unknown -- unit, root, census, cube, census of census, first related, census cube etc. with the related symbols. Tartaglia specifies that the name of census derives directly from Al Kwarismi, "perché così costumava Maumeth figliuolo di Moise della communa algebra inventore."

Gosselin summarizes Tartaglia but criticises the use of the term *radice*:

> Considéré que ce nom de Racine avec l'impropriеté qu'il a en nombres, a apporté aussi beaucoup d'obscurité en l'Arithmetique, & a retardé plusieurs gentils esprits, lesquels n'ont ozé s'y embrouiller, à raison que la chose leur sembloit si difficile de premier regard, tant pour l'impropriété de ce nom, que pour l'obscurité de ceux qui traitoyent de ces nombres en termes assez mal digerez...(***L'Arithmétique***, p.10v)

Here Gosselin seems to aim at the German cossist algebrists. Oronce Fine is in fact mentioned as the first author to have brought this doctrine to perfection,[110] followed by Peletier and Forcadel.[111] This is therefore an important step because it stresses Gosselin's relationship with the French tradition. This tradition, constituted by humanists interested in

110 With his work: ***Arithmetica practica***. Paris, S. de Colines, 1542. Conceived in a classical, and non-abacist, way, this very famous arithmetic is an indication of the new interest in the context of the Collège Royal and inaugurated the debate which gave new meaning to the distinction between theoretical and practical mathematics.

111 Pierre Forcadel. ***L'Arithmétique***. Paris, G. Cavellat, 1556-1557.

linguistic, orthographic, and rhetorical reform and in the popularisation of the sciences, gave great importance to terminology. Gosselin actually concludes:

> Or pour plus grande facilité, nous laisserons à part les Racines, plantes, & arbres, pour les iardiniers, afin que nous ne meslions les mechaniques avec les Mathematiques, & les choses terrestres auec les celestes; donc au lieu de Racine, nous dirons le costé, ainsi les costé Quarré, le costé Cubique...(p.11)

It would be difficult to imagine a thought which more effectively associates terminological precision with the distinction between disciplines and between theory and practice. We note also that Gosselin only later introduces the symbology for the powers of the unknown so defined. He is the pre-eminent algebrist in his field, and he can allow himself, from now on, deviate more freely from Tartaglia's original.

### *5. The Court and the Collège Royal: Conclusions*

We have seen how Gosselin explained the importance of algebra to Marguerite de France by mentioning its application to "vostre Astrologie." But already in the dedicatory letter to the first part of the ***Arithmétique*** Gosselin integrates it into a panoramic view of the mathematical sciences which stressed the role of the various disciplines, including those treated in the work. First among all the mathematical sciences appears algebra:

> Ne semble-il pas estre une chose totalement repugnante à la nature, que de dissoudre toutes questions proposèes, tant difficiles qu'elles soyent, & ce mesme d'une chose, qui ne peut estre, comme si elle pouvoit estre, et s'en servir generallement en toutes questions, & Problemes? entendre ce qui ne se peut faire, & ce que la Nature ne peut endurer, quelles choses sont toutes ces dignitez, qui passent le Solide, et toutefois par la vertu de ces Hypotheses &

> positions qui ne peuvent estre, venir finalement en la connaissance de ce qu'on demande? Cecy enseigne cette divine Algebre.

This constitutes the "marvelous" aspect of algebra. And that algebra is precisely the subject is clear from the comparison with a passage of the ***De arte magna***. For Gosselin writes:

> Algebra est numerandi scientia, quae docet ex falso verum elicere, & ex incognito quaesitum et cognitum deprehendere.

In these two passages Gosselin in fact makes use of that *topos* relative to algebra which Viète made famous: *nullum non problema solvere*.[112]

As for the role of algebra in scientific research, Gosselin had already referred to the theme of astronomy in the dedicatory letter of the First Part of the ***Arithmetique***:

> J'ay commencé par les nombres, c'est à savoir par l'Arithmetique & Algebre, lesquelles deux parties necessaires pour les Hypotheses de l'Astrologie, & pour le calcul des mouvements celestes, i'ay prins en main d'un Autheur qui a estè le plus fameux Arithmeticien, voire ie dy Mathematicien de toute l'Europe, lequel i'oze sans contredit appeller Prince des Arithmeticiens Praticiens: c'est ce grand Tartaglia, le los et renom duquel s'est espandu par toute l'Italie, de l'Italie est venu en notre France, & de la France a vollé par tout l'uniuers.(Epistre p.9)

Theories of the universe and predictions of celestial motions are therefore the "practical" purpose of the study of arithmetic and algebra. Indeed, the distinction between

112 Furthermore, like other algebrists of the time, Gosselin associates the very definition of algebra with the technique of false position. He had good reason to do so, since he had found the same idea developed in the work of Diophantus, whose work he was planning to edit.

astronomy as a contemplative science and as an art was a common topic of discussion throughout the period. Quite apart from that, however, astronomy was a science typical of the courts, as a theory of celestial motion, a means of compiling ephemerides for the determination of longitudes, and finally for the composition of almanacs with meteorological conjectures. How far this is from commercial problems!

Another topic to which both the princes and other nobles could be sensitive is that of the origins of algebra, which had relevance both for their own social standing, and for their interest in philology. This is a topic to which we will return in a separate chapter. Here, it is enough to notice that Gosselin insists on the Hellenistic origins of this discipline. He criticizes Tartaglia for not having mentioned the *auctores* of the discipline, but it seems that his criticism actually stems from Tartaglia's failure to trace a sufficiently *illustrious* genealogy for algebra. In fact, on this point Tartaglia mentions that Leonardo of Pisa learned algebra from the Arabs, and that Al Kwarizmi should be considered its inventor.

To conclude, Gosselin exemplifies the phase in which algebra had been sufficiently transformed as to be no longer identifiable with the abacus tradition. All of Gosselin's systematic effort is oriented towards the constitution of an algebra radically distinct from commercial, as well as practical, arithmetic. The most complete formulation of this attitude is still to be found in the ***De ratione***, in which he distinguishes between *rudior arithmetica*, i.e. arithmetic, not theoretical but only practical, and *subtilior arithmetica*, i.e. theoretical arithmetic and algebra, in which much space is given to the theory of equations.

We may now wonder why Gosselin chose Tartaglia as the basis of his first work. It is, however, not surprising if we consider that commercial arithmetic constituted the point of departure for those who dealt with algebra, whether at Court, in the academies,[113] or at the Collège Royal. Gosselin thought of writing an arithmetic. He was therefore inspired to translate an author who dealt in fact with commercial arithmetic, but stressed the role of algebra. The more precise the thought of the young Gosselin became, the more the distance between the two authors became evident. Finally the two approaches come into conflict. Tartaglia had integrated algebra into the *quadrivium* together with its original commercial context. In order to do so, he reaffirmed the connection between the speculative and practical parts of mathematics, with a reevaluation of practice in view of the "consideration of the natural philosopher." Tartaglia had conceived algebra as "practica speculativa." This move was striking for Gosselin because it reminded him of Ramus' view of the relation between theory and practice. The distinction was not between *scientia* and *ars*, but between the two different uses of the same discipline. But Gosselin did not want to simply reevaluate the arithmetic of the abacus schools. He wanted to make algebra into the most speculative part of arithmetic, without contacts with its original content.

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113 The connection between Gosselin and the Académie de Baïf and of Henri III has been shown by Natalie Davis in ***Mathematicians in the Sixteenth-Century French Academies: Some Further Evidence***, "Renaissance News" XI, 1958.

# Chapter Three

# Algebra

## Cardano's legacy in the manuals of Peletier and Gosselin

## *1. Introduction*

So far, we have seen that in a first phase our authors intended to inaugurate a new genre through which treatises of abacus mathematics were addressed for the first time to a French public informed by humanistic culture. Later, their goal was to make such mathematics into a "classic" discipline. We have also seen how their tactic was to exploit what had been done abroad, particularly by the Italians, in recent times. For Arithmetic, their main inspiration came from the Netherlands and Italy. For algebra, they drew from the German-speaking countries and Italy.

In considering this process for algebra, we shall give special attention to an Italian author, Cardano. Following this thread will help to determine Peletier's and Gosselin's most important innovations in algebra, as well as the aspects they borrowed from others.

The legacy of Cardano referred to in the title of this chapter might suggest a strictly disciplinary or retrospective approach which would be guided by a particular interest in the transmission of Cardano's rule for the resolution of cubic equations. That is not, however, our focus here. In fact, the dispute between Tartaglia and Cardano as to who had priority in publishing the solution of the cubics is one of the most famous in the history of modern mathematics. But to take this as a guiding question in reading Peletier and Gosselin would lead to disappointment. For neither author stresses nor develops this particular algebraic result in any important way. Yet, it would be a mistake to conclude that the absence of what we now take to be Cardano's most important contribution means that his algebra had not been received in France before Viète. Both Peletier and Gosselin, together with some German authors of algebra books in their time, mention Cardano at numerous points. First of all, in their historical introductions Cardano is cited as the author of the most recent algebraic manuals. Then, in the course of their texts, Cardano reappears as a reference for the formulation or solution of particular problems. As we shall see, these references occur with sufficient frequency for us to conclude that our authors relied heavily on Cardano as they worked out the presentation of problems in their own works. Therefore, in considering this different type of transmission, we will try to not limit ourselves by expecting only the specific and familiar sorts of indications of the reception of Cardano's algebra. Rather, we shall look for other ways in which Cardano was assimilated, thus paving the way for his further use in the algebra of the next generation.

### *2. The history of the second unknowns*

Within Cardano's algebra, we have chosen one discovery which he attributed to himself and which was ascribed to him by his contemporaries: the theme of the second unknown. We will study here its transmission from Cardano's texts to our French authors. This corresponds to what we would call "problems of the first and second degree in several unknowns," or "indeterminate problems of the first and second degree." These problems are expressible as a system of at least two indeterminate equations. The importance of the result I have taken for consideration here is evident insofar as it appears in the treatment of a large number of classical problems, both geometrical and numerical. In particular, we might recall that it was precisely through the renewed interest in indeterminate problems solved by means of indeterminate equations that, at the beginning of the next century, new results were obtained that constituted the so-called analytic geometry, in which a curve is expressed by an indeterminate equation.

Cardano deals with the theme of the second unknowns in the ***Practica Arithmeticae*** (1539), where he claims for himself the discovery of this rule. Of course, it should be clear that this is not a discovery in the absolute sense. Even without going back to the Arabic authors of the middle ages, we should recall that the *abaci* containing algebra might also include some references to problems involving a second unknown. We find this, in particular, in questions of "alligation", which took their name from the fact that they dealt with the value of coins that cast from various alloys. Thus, Luca Pacioli, following the tradition of the *abaci*, also precedes Cardano in these matters in his ***Summa de arithmetica*** (Venezia 1494), using the term ***quantita sorda,***[114] (in calculations, *quantita*, for short) as opposed to *cosa*, the first unknown. Cardano seems to have Pacioli in mind when he introduces his notation for the second unknown, i.e., 1 quan. Likewise, Christoph Rudolff writing in German in 1525 (***Die Coss***),[115] who broke with tradition by publishing the first manual devoted entirely to algebra, included a discussion of problems involving several unknowns.

In Cardano's fifty-first chapter ("*De modis omnibus imperfectis*") of the ***Practica arithmeticae*** we find a section entitled *"Regula de duplici"*:

> Quando aliquis ponit quaestionem in pluribus numeris & non nominat aliquem illorum tunc oportet uti regula quae vocatur de duplici etiam a me in lucem cum pluribus aliis edita et inventa. (***Opera Omnia***, Lyon 1663, p.86)[116]

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114 He also specifies: "Ora fa una nova positione per via de quantita sorda che li antichi chiamavano cosa seconda." ***Summa de arithmetica***, copy belonging to the Bibliothèque Nationale, Paris, Rés.V116, p. 192.

115 Christoph Rudolff von Jauer, ***Behend und hübsch Rechnung durch die kunstreichen Regel Algebre, so gemeinlich die Coss genennt werden***. Strassburg, W. Kopfel,1525.

116 Cardano continues as follows:"ut potestas aggregatum illorum numerorum 1 co. deinde per hoc investigabis summam illorum[.] post inventa summa quaeres per aliam positionem

In fact, the rule Cardano gives for this case is not quite a rule for using several unknowns, but rather a special case, arising as a way to solve problems by "iteration" of the process of assigning the unknown. In particular, the unknown is at first the square of an expression containing what will be the second unknown. But, since the procedure is expressed verbally, this does not require a new symbol. The next rule is the "*Regula de medio*." Leaving aside the search for generality, this rule is actually very close to what later authors will develop. This is the passage in which Cardano introduces the notation for the second unknown.

> Pone 1/2 quan. pro aggregato numeri quaerendi deinde divide 1/2 quan. in duas partes quarum una est 1 co. alia 1/2 quan. - 1 co. deinde multiplicamus invicem partes et productum fit 1/2 co. quan. m 1 ce. deinde operamur... (ibi.p. 87)

This is the beginning of a long and complicated rule (three times longer than our quotation) that applies for problems of the previous sort, for the exemple given is: "invenias duos numeros quorum multiplicatio invicem faciat 8 et quadrata iuncta sint cum ipsis numeris 27." In our terms, this means to find two numbers given their product and given the sum of their squares and the numbers. Cardano poses 1/2 quan. for the sum of the two numbers, so one number will be 1 co., and the other 1/2 quan. - 1 co. The solution goes on for about a page. Cardano concludes that this rule is more general than his own, and it comes from Magister Gabriele de Aratoribus, who got it from Luca Pacioli.

These two rules can give a sense of how far Cardano went in his "practical" treatise. There are other examples, to be sure, but reading Stifel one wonders why the German author is so certain of having found most of his matter on the second unknown precisely in

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unumquemque eorum per se et hoc modo in pluribus positionibus absolves quod in una fere esset impossibile."(ibidem)

Cardano, ie. in the ***Practica Arithmeticae***. For, the ***Ars magna*** would be more explicit on this topic, but was published only after Stifel's book.

The other examples containing second unknowns appear at various points within chapter fifty-one. But only the passage quoted above defines explicitly this type of solution, even though it is given in the form of an effective artifice. Earlier in the same chapter we also find a problem that Cardano will take up again, six years later, at the beginning of the section entitled "*De secunda quantitate incognita*" of his ***Ars Magna***. However, there is a significant difference between the two treatments, with greater systematic power in the Ars Magna. The ***Practica*** does not have the character of a theoretical work. It consists mostly of a collection of problems divided by topic, and a conclusion in which the examples are barely listed. All the theoretical points here are treated in a manner much reduced in comparison with what will appear in the ***Ars Magna***. In particular, Cardano does not take the opportunity to synthesize in a single, properly stated rule, which is to say in a theoretical formulation, the solutions proposed to various problems. Rather, he simply describes the class of problems and then addresses himself to some of them separately by means of examples in the collection of problems that constitutes the final chapter. This concluding chapter, "*De quaestionibus Arithmeticis super capitula praecedentia*," takes into consideration a large number of problems (about fifteen) solvable by means of the second unknown. Now, almost all of these examples were later taken over by Stifel in his ***Arithmetica integra***. One see this directly in the chapter on the second unknowns, which he defined as *secundae radices*, or in the section dedicated to problems relative to that chapter or, finally, in the appendix devoted to Cardano.

In connection with the *secundae radices* Stifel writes:

> Quando in pronunciatione alicuius exempli, post positionem 1 occurrit adhuc alius numerus absconditus, sub indeterminata proportione ad numerum latentem sub 1 prius posita, tunc ponitur numerus ille absconditus sub 1A.(p.251v)

After about one page, he concludes:

> Reliqua vero quae docenda sunt de radicibus secundis, dicam per occasiones ponendorum exemplorum. Christophorus et Hieronymus Cardanus tractant radices secundas sub vocabulo Quantitatis ideo eas sic signant 1 q. Latius vero eas tractavit Cardanus. (...) Eas autem Cardanus pulchris exemplis notificavit

Further, in the chapter that brings together additional problems related to the previous chapters, Stifel writes:

> Incredibile est, quam late vagetur secundarum radicum usus, quarum exempla, ordo et ratio dicendorum, nunc requirit.(p.292)

This is followed by a long series of examples, some of which are taken from Rudolff, but the majority of which come from the ***Practica Arithmeticae***. Since we cannot give full consideration here to the development of Stifel's treatment of these examples, and so underline the affinities and differences between the two authors, I shall limit myself to referring the reader to the two pages reproduced in the Appendix to this chapter in order to give at least an example of solution by second roots. It is important, however, to stress that Stifel succeeds in giving a much clearer and better-defined presentation than those in the analogous passages by Cardano in the ***Practica***. Stifel's unique improvement follows from his introduction of the first capital letters of the alphabet for unknowns after the first one.

Finally, Stifel devotes an entire appendix to a return to some problems of Cardano, giving his reasons in the Dedication to "Moecenatem Adolphum Glaubruck, Francofordiensem Patricium":

> Arithmeticam Hieronymi Cardani talis est, mi domine Adolphe, ut sese tibi satis abunde sit commendatura, dum ea legeris. Habet enim multa rara, quae

> alibi nusquam legimus. Delectant autem quaedam exempla eius adeo, ut quiescere non potuerim, donec ea tibi peculiariter rescripta mitterem. Oro te nihilominus, amore artium, quas tantopere colis, quatenus eam Arithmeticam totam legas a principio diligentissime, et assuescas, signa eius, quibus ipse utitur, transfigurare ad signa nostra. Quamvis enim signa quibus ipse utitur, vetustiora sint nostris, tamen nostra signa (meo quidem iudicio) illis sunt commodiora.(p.3O6)

This quotation shows to what extent Cardano's treatment was interesting for Stifel, but also his certainty that he had improved on Cardano from the point of view of notation. One must note, moreover, that the majority of the problems included in this appendix (thirteen out of seventeen) deal with the second unknown. This preponderance of problems of the second unknown confirms that Stifel considered this to be one of the most important themes, and Cardano the principal authority in this regard. Stifel's judgment provides yet another index of the importance accorded Cardano by the algebrists of the period.

### *3. Peletier's algebra*

Peletier published ***L'Algèbre*** in Lyon in 1554,[117] fifteen years after the publication of Cardano's ***Practica Arithmeticae*** and nine years after the publication of the ***Ars magna***. At first glance, one could say that the principal influence of Cardano was by way of Stifel, and hence from the ***Practica***. In fact, Peletier confers on his manual the same structure as Stifel's work, although in a much reduced form, excluding in particular the section devoted to the tenth book of Euclid. The most important difference, of course, is that Peletier's book is entirely devoted to algebra. Stifel's influence on Peletier is also suggested in the first important essay on this text,[118] but we have already seen that, on the topic which concerns us

117 For the success of the ***L'Algèbre*** and its various editions, see the first chapter.

118 See Bosmans, 1907.

more particularly, Stifel considered himself dependent on Cardano.

But before entering into the matter of the second unknowns, it will be useful to have an idea of the mathematical content of Peletier's book, leaving the issue of his choices in genre and style for the fifth chapter.

Peletier states in his second chapter, *Des Nombres appartenans aus operacions de l'Algebre*:

> Combien que l'Algèbre mette generalmant an operacion toutes sortes de nombres: touttefoes elle considere principalement les nombres Radicaus, c'ét a dire qui ont en eus quelque Racine a extrere. Car la perfeccion de l'Algebre, git an l'invencion des Racines, soet Racionnalles ou irracionnalles. (p.5)

Here Peletier had not yet introduced the notion of equation. So it seems that he approches this notion by following a gradual path. Given that the reader expects algebra to be an "extended arithmetic", he can rely on the numerical intuition of the solution for equations involving powers, i.e. the extraction of root. But this is only the first step. He then introduces the symbols for cossic numbers, which imitate those given by Stifel, but with some modification:[119]

119 We may add in passing that Peletier adopts Pacioli's and Cardano's sign for the plus and the minus, while Stifel used the signs with which we are now familiar.

This notation requires some skill, because the character does not make explicit the number corresponding to the power. Peletier therefore explains how to determine that number. Then he devotes a chapter (starting p.11) to the "numbers which belong particularly to algebra" (as opposed to the larger category mentioned above): 1) the "nombres Denommez ou Cossiques", which are before a cossic sign, such as 5 . 2) the irrational numbers, such as 20, which come after a cossic sign. 3) the numbers preceded and followed by a cossic sign, such as 20 . The first book is devoted to the first type, the second to the other two. Thus, Peletier gives the four operations for cossic numbers.[120] The absolute numbers are the known terms, whereas "absolus des cossiques" are the coefficients.[121]

Then Peletier introduces a new notion, that of equation. The title of the section is, significantly, *De l'Equacion, partie essancielle de l'Algebre*. Peletier writes:

> L'Equacion e l'Estraccion de Racines, sont deus parties de l'Algebre, equelles consiste toute la consommacion de l'Art. Pource, nous les tretterons toutes deus clerement, e au long. Par ce moyen nous reduirons toute l'Algebre a une simplicite tele, que de tantr de regles qu'an ont fèt les autres, nous n'an ferons

120 Which he calls *Algoritme*, according to the medieval tradition.

121 Bosmans has a list of Peletier's terminology, which now needs some correction. This topic, which unfortunately cannot be pursued here, is of intinsic interest, since Peletier's practice was widely imitated.

> qu'une seule, qui les comprandra toutes, einsi qu'a fèt Stifel. Equacion donq, ét une equalite de valeur, antre nombres diversement denommez. Comme quand nous disons 1 Ecu valoer 46 Sous (...) einsi, quand nous disons, 1 egal a  : il y a une Equacion antre 1, avec sa denominacion de  e 4 avec sa denominacion de  de sorte, que si  vaut 16: il faut que  valhent aussi 16. (p.22)

But the equation is built on a problem or, in Peletier's terms, on a question, by making use of the known numbers. To put a problem in the form of equation is in fact considered a particular technique:

> premierement, il s'antand essez, que les nombres exprimez es Questions sont ceus qui nous guident: et par l'eide dequez nous decouurons les Nombres inconnuz. Il faut donc an cete Question proposee, que par le moyen de 10, Nombre exprime, se trouue celui que je demande.

After having explained some points concerning the simplest case, when the equation is between two fractions, Peletier goes on to explain what he calls the extraction of roots, and gives a procedure for it.[122]

The procedure is not unusual, but it deserves attention because it is the main technique of the first book. It goes as follows:

*step one:* take half of the coefficient of the unknown, with its sign;

*step two:* square the result of *step one* and add it to the known term. If the known term is negative, subtract it from the square of the result of *step one*;

*step three:* take the square root of the result of *step two* and add it to the result of *step one*.

122 In the chapter "De l'Extraccion artificielle des Raciens des nombres Cossiques Composez e Commecomposez, a la forme des nombres Absoluz."

For instance, to take the "*racine censique*," i.e. the square root of 6 + 16 . From *step one* we get + 3; from *step two* we get 9 + 16 = 25; from *step three* we take the square root of 25 from which we subtract + 3, and we get 8, the square root of the given "*nombre cossique composé*" 6 + 16 . Of course, the procedure can be applied to more complicated cases. But the reader should first learn the technique on "*nombres censiques composés et commecomposés*," ie. on binomials (or polynomials) and then apply it to solve equations.

Peletier ascribes this procedure to Stifel. However, he states that the distribution of the matter is entirely his, and in fact we see that the procedure itself is stressed more than in Stifel's book. What does this procedure consist of? We should assume that, for Peletier, in the normal form of an equation the first member consists of a power of the unknown and the second member is an algebraic expression. If we look for the unknown quantity, its is necessary to extract the root of that second member. Given that he deals mostly with second degree equations, they will be for the most part binomials. Thus, the first thing to learn is how to extract the square root of a binomial, a process which will then be applied to equations and give the solution of second degree equations.[123] In fact, the structure of the text itself is built on this discovery, insofar as the book is not centered around the degrees of the equations, but around the use of the procedure.

This procedure on binomials precedes the section in which Peletier introduces a new technique. This is applied in order to determine the roots of an equation in terms of coefficients. Peletier himself, usually very careful about attributions, presents this as his

123 The application of the procedure for the extraction of roots is what Bosmans identifies immediately with the determination of a solution formula for second degree equations. In fact, this interpretation prevents Bosmans from reconstructing exactly Peletier's succession of topics, so that his description of Peletier's book does not show a careful plan. Techniques that can be "seen as" or "shown to be" equivalent mathematically cannot necessarily be treated as equivalent by the historian.

own invention. In fact, he presents these techniques as applications of the extraction of roots to particular (and particularly easy to deal with) cases. In this matter, he goes far beyond his (printed) predecessors, by systematizing and extending some earlier techniques. Let us follow him through the first example:

1 egal a 12 m. 36 $[x^2 = 12x - 36]$

Peletier writes that the square root of the "censic" [square] number 36 is the same of the root of the binomial, i.e. 6. He gives no more explanation, but this result is obtained by applying the three previous steps: *step one)* 6; *step two)* 36 - 36; *step three)* 0 + 6.

Of course, we only get one (double) root. In fact, we only accept positive roots. Furthermore, only roots greater than one are actually taken into consideration. Peletier's procedure allows him to unify in one rule the series of "canons" or single rules of solution for the different cases of second degree equations. Furthermore, when he had studied the known term more carefully, he saw it as the product of the roots. Therefore, he goes beyond Stifel's rule "amasias" in his suggestions for the study of coefficients.[124]

Only at this point does Peletier feel that the reader is ready to understand the statement of "*la grande reigle generale de l'algebre*":

> Au lieu du Nombre inconnu que vous cherchez, metez 1 : Avec laquele fetes votre discours selon la formalite de la Question proposee: tant qu'eyez trouué une Equacion convenable, e icelle reduitte si besoin èt. Puis, par le Nombre du sine majeur Cossique, divisez la partie a lui egalee: ou an tirez la Racine tele que montre le Sine. E le Quociant qui prouiendra (si La Division suffit) ou la Racine (si l'extraccion èt necessere) sera le Nombre que vous cherchez.

Then Peletier gives a series of examples, of gradually increasing difficulty, intended to teach manipulations of the equations such as division and reduction. The problems, it is

124 For an account of Stifel, see Troepfke.

worth noticing, have to do with numbers or commercial cases.

Let us now take into consideration our main theme, the second unknowns. After some pages devoted to the algorithm of the second unknowns, which is to say to the four operations performed with them, Peletier provides five examples, of which three are taken (according to him) from the ***Practica arithmeticae***, while one is taken from Stifel. This is the transmission of the theory of the second unknown from Cardano to Stifel to Peletier. But Cardano, in the meantime, had published the ***Ars magna*** where, as we know, he developed more definitively the doctrine of the second unknown in the ample space he allowed for that purpose. In this work he treated systematically many indeterminate problems of the first degree, but also some of the second degree, although of course without a unique formula of solution. What is systematic, rather, is the series of problems indicated, and Cardano's use of a relatively uniform method, although subject to minor variations.

It will no longer be surprising to see that in fact Peletier innovates with respect to Stifel, and does so by making use of Cardano' ***Ars Magna***. Peletier gives the following definition of the second unknowns, which he calls, following Stifel, *secondes racines*:

> Les Racines Secondes viennent en usage quand deux nombres ou plusieurs se proposent, entre lesquels ne se fait aucune comparaison expresse par addition, multiplication, division ou proportion, par difference, ni par Racine: qui sont les cinq manières de comparer les nombres ensembles. Desquelles la proportion est la principale, car les autres seules bien souvent n'excusent pas l'usage des Secondes Racines.(p.96)

Now, this passage clearly retraces the lines of a similar passage in chapter XI of the ***Ars magna***, and shows us that in fact Peletier also adopted Cardano's theoretical point of view with respect to the second unknown:

> Solemus autem his uti positionibus, cum duorum numerorum, qui ab initio ponuntur, nulla exprimitur comparatio, nec in aggregato nec in differentia,

> nec in multiplicatione, nec in divisione, seu proportione, nec in radice, his enim quinque modis comparantur numeri, quare si unus consistat, nulla est secundae quantitatis utilitas, sed una positione quaestio solvitur. (***Opera Omnia***, p.244)

We may note in passing that we have here the definition of a functional relation, as opposed to that of a determinate operation. Of course, the use of the *racines secondes* allows one to perform, through the solution of the system, just those operations that would be indefinite.

Peletier's version of the problem that opens the chapter entitled "*De secunda quantitate incognita*" of Cardano's ***Ars magna*** is what we are going to see here.

Cardano writes:[125]

> Up to this point we have been treating of new discoveries quite generally. Now something must be said about certain individual types. It frequently happens that we must solve a given problem by using two unknown quantities. There follows an example of this which we could otherwise explain only with difficulty. Three men have some money. The first man with half the other's would have had 32 *aurei*; the second with one-third the other's, 28 *aurei*; and the third with one-fourth the other's 31 *aurei*. How much had each? (Ch.IX,p.71)

There is no need to insist on the fact that Cardano is not ready to give a general rule. The next sentence gives the procedure of assignment of the unknowns:[126]

> We let the first unknown thing to be the first man's share, the second unknown thing to be the second man's share; thus for third man there will be left 31 *aurei* minus one fourth of the thing and one fourth of the quantity. (***Opera Omnia***, p.241)

Cardano pursues his calculations on this basis for a couple of pages. As we can seen the third unknown is defined in terms of the other two. Peletier repeats Cardano's statement and

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125 We use here the recent translation by T. R. Witmer.

126 Here we translate form Latin, because Witmer adopts his notation directly at this point, and this would obscure matters in our context.

gives a first version of the solution following Cardano's reasoning, even though he adds some explanatory comments. At the end he adds some very inspiring remarks:

> An cet Example, j'è suivi de point en point la proposicion e la disposicion de Cardan. An quoe j'è eté aussi long comme lui, e un peu plus cler. E n'ut eté pour montrer la singularite de l'Algebre, e comme elle git an discours, e comme elle exerce les espriz: j'usse lessé cete explicacion sienne, laquele il appelle facile, pour an mettre une autre qui s'ansuit, de notre dessein.

Cardano's solutions gives Peletier the opportunity to state that algebra is explicit reasoning. But the next sentence starts the solution procedure: "Le premier a 1R . Le second, 1A. Le tiers, 1B." Because of the condition on the first,[127] 1R + 1/2 (1A + 1B) = 32. By reduction and "transposicion", 2R + 1A + 1B = 64. Peletier calls this the *fir*

*st equation*. Because of the condition on the second, 1A + 1/3 (1R + 1B) = 28. Thus, 1R + 1B + 3A = 84, *second equation*. Because of the condition on the third, 1B + 1/4 (1R + 1A) = 31, hence 1R + 1A + 4B = 124, *third equation*. We now **add the third to the second equation**, and we get the *fourth equation*, i.e. 2R + 4A + 5B = 208. We now can **subtract the first equation from the fourth**, and we get the 3A + 4B = 144, the *fifth equation*. Let us **add the first and the second equation**, we get 3R + 4A + 2B = 148, which is the *sixth equation*. By **adding the first and the third equation** we get the *seventh equation*, i.e. 3R + 2A + 5B = 188, whereas **by adding the sixth and the seventh equation** we get 6R + 6A + 7B = 336, which is the *eighth equation*. Let us now multiply the third equation by 6, getting 6R + 6A + 24B = 744, i.e. the *ninth equation*. Given that the first two terms of equation eight and nine are equal, we can write 17B = 408, and we obtain the third number, B = 24.

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127 Here I shall adopt, unlike Peletier, our signs for +, - and =, where he has p. m. and egal. I also introduce the brackets, for typographical convenience.

Now, because of the fifth equation,[128] 3A + 4B = 144, hence 3A = 144-96, we shall have 3A = 48, i.e. A = 16, the second number.

Because of the first equation, 1R + 1/2 (16+24) = 32, so 1R = 32 - 20 = 12, the first number.

Peletier concludes: "*Ce discours est trop plus facil que l'autre. Mes il fet bon voer deus inuancions an meme intancion*."

This procedure might not look brief to us, but it was remarkably short if compared to Cardano's procedure.

What did change? The main innovation is the introduction of as many symbols as there are unknowns in the problem, and the unknowns in the problem coincide with the unknowns of the equations. Besides, Peletier is very systematic in structuring the solution through the various transformations of the first equations, those which, for us, belong to a "system." Then he uses, as Cardano does, the method of addition and subtraction of equations. However, in this too he has a happier hand than Cardano because he does not introduce the arbitrary coefficients which give a very artificial impression to the earlier writer's text. Peletier's innovations made the solution simpler and shorter, and the notation, though inspired by Stifel, points, in his use, to the path later taken by Borrel,[129] Gosselin and then Viète: to make use of a sequence of the first letters of the alphabet.

It should be remembered, moreover, that between Cardano and Peletier there came

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128 Peletier writes "3A p 3B etoet egales a 144": the second term is a misprint and should be 4B; the next passages do not carry over the mistake.

129 Jean Borrel, also known as Buteo, author of the ***Logistica***, had a theoretical dispute with Jacques Peletier on the latter's translation of Euclid (see chapter 1). In fact, his algebra would deserve a more extended treatment.

another important writer on algebra, Johann Scheubel,[130] who devotes two pages to the *secunda quantitas*, with notation in the style of Cardano. He is known by Peletier, who mentions him in the introduction. However, his contribution for our purpose is in the structure of the work, based on the degree of the equations. Otherwise, he was not influential in France: e.g. his notation for plus and minus, i.e. + and - , was not adopted in France until some time later.[131]

### *4. Gosselin's <u>De Arte magna</u>*

Now we turn to Gosselin's ***De Arte magna seu de occulta parte numerorum quae algebra et almucabala vulgo dicitur***. Without a doubt, the title of this work brings together references to the algebraic manuals of Peletier and of Cardano.[132] Nonetheless, the influence of these two authors is of two rather different kinds. On the one hand, Gosselin takes Peletier as a precedent in his explicit formulation of some general theoretical and philosophical preoccupations. These are entirely absent from Cardano. On the other, it is

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130 Johann Scheubel, who dedicated his most famous book, ***Euclidis megarensis sex libri priores*** to Anton Fugger and to the sons of Raimond Fugger, was *professor ordinarius* at Tübingen. His algebra manual, ***Algebrae compendiosa facilisque descriptio qua depromuntur magna Arithmetices miracula***, was first published as a preface to the Euclid, and then separately, in Paris, in 1551 by Cavellat, who was also the publisher of ***De occulta parte numerorum*** by Peletier (156O). On this author, see in particular Mary S. Day, ***Scheubel as an algebraist***, Teachers College, Columbia University, N.Y. 1926.

131 It would be interesting to determine more about the use of this text at the Collège Royal. For, its publication, says the publisher Cavellat, was requested by the *lecteur royal* Magnien (see reference in Chapter 1)

132 As we have seen, the title of the latin version of ***L'Algèbre*** is actually: ***De occulta parte numerorum, quam algebram vocant, libri duo***.

clear that Gosselin has adopted the scheme for the structure of the work from Cardano's ***Ars Magna***, slightly modified, rather than that of Peletier. In fact, Gosselin structures his treatise around the classification of equations on the basis of their degree, just as Cardano had done in the ***Ars Magna***. In so doing, he declines to follow the alternative models provided by Stifel and Peletier, and well as the ***Practica*** of Cardano himself, which would have led him to proceed either from the distinction between the algorithm and the "extraction of roots" of binomials for rational numbers, or from the algorithm and the extraction of roots for irrational numbers.

In fact, Gosselin's theory may be distinguished in three ways from the theories sustained by previous authors, all of which lead to the theory of equations. The first, already mentioned above, is the structure of the work; the second is the influence of Borrel and Nunez; the third is the study of the ***Arithmetica*** of Diophantus. Gosselin was in fact the principal expert on Diophantus in Paris. We have seen in the first chapter that he made public a number of times his intention to issue an edition of the ***Arithmetica*** together with an extensive commentary. Gosselin argued that this was much needed since the commentary done by Planudes and the edition done by Xylander (Basel 1575) had raised more questions of interpretation than it had resolved. Yet, Xylander's influence is strong in the notation and in the names used for some Diophantian procedures, as we shall see.

As to the notation, I think we should notice that it shares one feature with some Italian notation (in particular, Cardano's), i.e. it is very compatible with printing, more than the one proposed by Peletier. We can therefore give it directly as:[133]

1 L. Q. C. QQ. RP. QC. RS. QQQ. CC.

133 I should stress that Gosselin makes a mistake here, and forgets the eighth power, QQQ, or 256.

It is important to notice that Gosselin's nomenclature for the powers of the unknown seems at first to depend on Xylander's notation,

N. Q. C. QQ. QC. CC.

however, we see that Xylander adopts Diophantus' additive structure on the exponents, whereas Gosselin keeps the multiplicative structure on the exponents, in accordance with the Italian algebraic tradition. Gosselin is aware of differing from Diophantus on this point.[134] Furthermore, Gosselin uses P and M for + and -. This is again a concession to printing (if compared to the previous p and m). There is no sign of equality. Yet, Xylander had +, - and the vertical equality. Before looking at Gosselin's treatment of the second unknowns, we shall give a short account of the main aspects of the mathematical content of ***De Arte magna***. Gosselin announces in the dedicatory letter that he was to "demonstrate" this part of mathematics, for he had taken as his aim to demonstrate mathematically any part of mathematics. The term "demonstrate" seems to refer to a magisterial exposition of the topic, so that it can be taken to suggest that algebra was becoming a subject in the colleges, and that Gosselin was engaged in presenting it *adolescentibus discendi cupidis*.[135] However, in Gosselin's case, it should be taken as an indication of the fact that he intended to produce more "demonstrations" or "proofs" for algebraic procedures.

After dealing with the algorism, with extraction of roots, ratios and proportions, Gosselin explains the rule of three (*Datis tribus numeris, quartum proportionalem reperire*) and the two rules of the simple and of the double false position. It is the rule of false, called also of false position, that Gosselin calls the rule of the simple or double hypothesis. It is

134 See Klein p. 274, and Gosselin 1577, p. 4v.

135 This is the phrase used by Gosselin in the dedicatory letter (f. a iiii).

interesting that he takes this topic into consideration *before* dealing with equations, thereby reversing the order of Cardano's ***Ars magna***. In fact, here the rule is taken as in the arithmetic treatises, i.e. as applying to proportions. Only later in the text, in book 3, we shall see the same rule applied to the second degree. But this will be the *fictitia aequatio*, taken from Diophantus.[136] Peletier had excluded the rule from his ***L'Algèbre***, but he had dealt with it in ***L'Arithmétique***, where he, unlike Gosselin, gives a very explicit definition:[137]

> La Regle de Faux, que les Arabes appellent la Regle Catain, est ainsi ditte, parce que d'un cas faux presupposé, elle enseigne a trouver le vrai. Et est celle de toutes les Regles vulgaires, de laquelle l'usage est plus beau et plus ample. Elle a deux parties, l'une d'une seule Position fausse, l'autre de deux. La Regle de Faux d'une Position a presque pareille operation a celle de la Regle de Trois, excetté qu'en la Regle de Trois nous avons trois termes cognuz: ici nous n'en avons qu'un (i'entens qui viene en operation) a la semblance duquel nous en formons deux autres, l'un multipliant, et l'autre divisant.

In other words, we establish a proportion, in which one ratio is given and the other is between the unknown and an arbitrary quantity. By the rule of three, the unknown is determined. As to the double rule of false, Peletier writes:

> La teneur de la Regle par deux fausses positions est telle. Au lieu du Nombre de la question incognu que vous cherchez, empruntez un nombre à votre plaisir: et par icelui faittes votre discours selon la formalité de la question, tout ainsi que si c'etoit le vrai Nombre que vous voulez trouver, e si voiez que n'aiez trouvé votre point, notez le Nombre, et semblablement a coté de lui, la difference en laquelle vous avez failli avec son signe de Plus ou de Moins. Apres, empruntez un autre Nombre, par lequel faittes

136 As we shall see in the following chapter, there is another form of equation which derives from the rule of false position and from Diophantus' use of it, the one Xylander and Gosselin call *adaequatio*.

137 We prefer to adopt Peletier's definition instead of translating into our terms. In other words, we make the false assumption that a certain value is a solution. Even though this is in general not true, to replace such a value in the equation will allow us to establish a proportion which will give us the true solution. It would be important to compare the various uses of the term, especially for cases like Gosselin and Stevin, then Bachet, with the Arabic tradition of the abacus combined with the new reading of Diophantus. For a discussion in connection with Fermat, see Mahoney 1973, p. 156.

> semblable discours: et si par icelui n'avez non plus trouvé ce que cherchez, notez encore celui Nombre, et semblablement à coté de lui, la difference, avec son signe de plus ou de moins. Apres multipliez le premier Nombre emprunté par la difference du second par la difference du premier (et cela est perpetuel) et gardez les deux Produitz. Puis si les signes sont pareils, c'est à dire tous deux de Plus, ou tous deux de Moins, otez le moindre produit du plus grand: et semblablement otez la moindre difference de la plus grande: et par le residu d'icelles, divisez le residu des Produiz: le Quotient sera le vrai Nombre que vous cherchez.

About the simple hypothesis, Gosselin writes that the rule prescribes to introduce a new number, which is false, but such that the error can be corrected. The process can be expanded to the rule of the double hypothesis:

> Si pro ignota quaestionis alicuius quantitate, duae quaelibet eiusdem generis assumantur, et ex utraque sigillatim quaestionis formula pertractentur, si quid vel supersit demum, vel desit, cum nota redundantiae, vel defectus ascribatur, erit sicut differentia hypothesium ad errorem eius hypothesis; cuius erratum operis secundum proportionale est assumptum, quod hypothesis erratum, hypothesi vel additum, si quidem hypothesis fuerit minor quam oportuit, vel deductum si maior, quaesitam suppeditat quantitatem.

We see that Gosselin, unlike Peletier, does not give instructions, but explains the structure of the procedure. Gosselin is actually writing a theory and devotes about twenty pages to the rule of the double hypothesis. Through theorems and examples he develops a practice, if not a theory, of errors which can allow the arithmetician to find the true solution. It seems that Gosselin considered learning this technique as absolutely crucial for the subject, thus at the center of his elementary treatment of algebra. It is likely that Gosselin considered the rule of false and the *fictitia aequatio* so important because of Diophantus. So, what had been excluded from algebra by Peletier, i.e. the rule of false, becomes a crucial part of the book.

Of some interest are his demonstrations, on the basis of Euclid, of the two forms of the rule, as well as some preliminary theorems which he then uses to apply the rule to a geometrical case, the invention of two mean proportionals. This problem is what captured

Montucla's attention, as he writes "*J'ai idée d'y avoir vu anciennement des essais assez ingénieux d'application de l'algèbre à la géométrie*."[138] Bosmans[139] does not seem to grasp the sense of this section, because at first he appears surprised that all the examples are cases in which the false position holds. This is clearly Gosselin's aim, given that he has demonstrated it previously. Secondly, he does not see the interest of the geometrical example in Gosselin. In fact, it is important insofar as Gosselin goes beyond his favorite predecessors Nunez and Tartaglia and develops connections between algebra and geometry by applying algebra to a problem instead of simply using Euclid's authority in proving algebraic results.[140]

Thus far, Gosselin explains at the beginning of Book 2, he has dealt only with those parts of arithmetic which are needed to do algebra. The purpose of Book 2 is to give an algorism of the "*nomina vel quantitates*", ie. of monomials. Besides the operations, Gosselin gives the rules for the signs.

Book three is devoted to equations. Gosselin states that what preceeds has this section as a goal, whereas this topic is of value in itself. So he passes to equations "tanquam ad apicem et fastigium huius scientiae." Equation is defined as "duarum quantitatum diversi nominis et valoris ad unam aestimationem reductio", a definition very similar to the one given by Peletier's.[141] Yet, very soon it appears that equations are conceived in a new way.

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138 Montucla, tome 1, p.613.

139 Bosmans, pp.51-52. Gosselin 1577, p.35v-39.

140 Given that this passage is richly annotated by the unknown reader of the copy present at the Bibliothèque Nationale, I shall develop a presentation of Gosselin's treatment and a critique of Bosmans account on this point.

141 Peletier had "*Equacion èt une equalite de valeur, antre nombres diversement denommez*."

This happens first of all through the classification by degrees, much more explicit than in Peletier's text. Secondly, through the classification of the methods used by Diophantus, listed as a types of equation with the names invented by Xylander.

The "canons" or solutions of first (*aequatio simplex*) and second (*aequatio composita*) degree equations are not particularly innovative, but we must stress that they are presented in a short and effective way. These solution formulas are actually demonstrated, also thanks to explicit "axioms" recalled at the beginning of the section.

By the *aequatio ad hanc proportionalis* Gosselin means biquadratic equations, and he deals with them in chapter 5. After talking about *aequatio tertia* i.e. third degree equations, which we shall discuss further, Gosselin introduces the first kind of equations taken from Diophantus' *Arithmetic*, the *fictitia aequatio*:

> Non est vulgaris operae aut mediocris ingenii aequationem sibi effingere, cum nihil est quod facta ratiocinatione aequale sit residuis quantitatum speciebus, et quod mirum est eam infinitis variare modis, quod nos Diophantus docet in Arithmeticis.

Gosselin's first example is the following. Let 6Q + 16 = "a certain square." The solution is derived from the famous problem II, 8 of Diophantus *Quadratus numerus propositus, dividatur in numeros duos quadratos*,[142] and the following. We shall see that here Gosselin uses one of Diophantus' methods to make a determinate equation out of an indeterminate.

This is his procedure. *Effingemus* a square the root of which is a monomial containing L. The condition on this is that the known term should be not greater than the root of the known term in the equation. Let us take 2L + 4. Its square will be equal to 6Q + 16. So we get L = 8. Gosselin, unlike Diophantus or Xylander, tests the procedure with another "made up" square, ie. the square of 3L - 4 . In fact, by equating it to 6Q + 16 we get 3Q - 24

142 See Xylander p. 44.

"*aequalia nihilo*."[143] Among the examples, Gosselin gives one which uses again problems 8 and 9 of Book 2 of Diophantus, i.e., "given a square, find the common difference of an arithmetic progression in which the given square is between two squares."[144] The reciprocal of this problem will be the example in the next section, using what Xylander and Gosselin call the *duplicata aequatio*. The new problem is "given the common difference, find three squares in arithmetic progression."[145] To define this type of procedure, or, as he puts it, of equation, Gosselin writes:

> Est et alia ratio a Diophanto excogitata cum aequatio non potest per praecedentem stare, quando nimirum duae quantitates supersunt quarum utraque alicui sit aequalis, ut sint 1LP2 aequalia Quadrato, et rursus 1LP3 aequalia quoque Quadrato.

Here again Gosselin starts with Diophantus' example, in this case taken from[146] problem 12 of Book 2: "To add a number to two given numbers, so that each of them makes a square." Here Gosselin paraphrases Diophantus. The two numbers are taken to be 2 and 3, and the number to be added is 1L, hence 1L + 2 will be equal to a square, and 1L + 3 will be equal to a square. Xylander translates the following sentence as "Hoc genus vocatur duplicata aequalitas." We take into consideration the difference between the two numbers (in this

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143 I mention this phrase because it is one of the rare instances, in this period, of equations in which the second member is zero. There are about three instances in Peletier's text, and a couple in Gosselin's. Clearly, however, this aequation is obtained by elimination of the second member. The canonical form is, for Gosselin, with the monomials in the first member and the known term in the second. This is different with respect to Peletier, who had in the first member only the monomial of higher degree.

144 In fact, Gosselin's phrase is in terms of congruent numbers in the ancient sense: "Dato Quadrato, reperire numerum Congruentem, sit datus Quadratus numerus 100".

145 Again, Gosselin's words are different ("*Dato numero Congruente reperire Quadratum*"), even though he actually finds three squares.

146 Here we follow Xylander order. In VerEcke's version, this problem is indicated as problem 11.

case, 1), and we look for two numbers the product of which is this difference. Diophantus, and Gosselin, take 4 and 1/4. The procedure is then to take the square of half of their difference [here, $(15/8)^2 = 225/64$], and to set it equal to the smaller square, i.e. 1L + 2. Thus we get 97/64. Otherwise, we take the square of half of their sum (here, 289/64) and we set it equal to the greater square, i.e. to 1L + 3. Hence, we get the same 97/64. Gossselin concludes:

> Huius rei demonstrationem remittemus ad nostras in Diophantum animadversiones, in quibus quaecunque vel mutila sunt vel certe ab interprete non considerata, tum restituemus, tum vero Deo iuvante demonstrabinus omnia.

Here we confirm what we discussed in chapter 1: Gosselin intended to write a commentary on Diophantus which should contain demonstrations for the passages not clarified by Xylander and Planudes.[147]

Let us now return to the special problem to which Gosselin applies the *duplicata aequatio*. Gosselin is very proud of having found the solution, which Fibonacci, Pacioli, Tartaglia, Cardano and Forcadel were not able to find. But -Gosselin writes- they did not know what was hidden up to these times; it seems that Gosselin thought precisely of Diophantus. The solution, in fact faithfully follows the procedure just explained. We are given the common difference 96 (*numerus congruens*): if we the call Q the smallest square, there exist two squares, 1Q + 96 and then 1Q + 192. In order to apply the procedure, we see that 96 is the difference between the two known numbers. Now we look for numbers the product of which is this difference: 4 and 24; 6 and 16; 8 and 12. Now we must take the square of half the sum or the difference of two numbers and set it equal to the greater or,

147 This of course does not mean that he did not also want to prepare an edition of Diophantus, especially at a later date, in 1583, as we shall see in the next chapter.

respectively, the smaller number. Gosselin tries first with the 8 and 12, but he gets a negative value for Q, therefore, he tries again with 4 and 24 and succeeds: $(1/2\ (28))^2 = 196$, so 196 = Q + 192, Q = 4. The arithmetic progression of squares will then be Q , Q + 96 , Q + 192 i.e. 4, 100, 196. Bosmans does not seem to follow Gosselin and declares that after the decomposition of 96 into factors "[Gosselin] abrège et semble opérer quelque peu au hazard. il était bien aisé cependant d'achever la solution comme elle était commencée."[148] Bosmans has not recognized in this instance Diophantus procedure. There is no "hazard", but just the application of the previous procedure. Clearly, however, Gosselin does not accept negative solutions.

These are the main passages of Gosselin's work in which he makes explicit use of Diophantus' ***Arithmetic***, and in which this leads him beyond his predecessors. In fact, there is a more implicit use. Diophantus made Gosselin rethink the whole theory of equations in several unknowns. The ***Arithmetic*** is crucial to our present topic insofar as it brings together a large number of indeterminate problems. However -and we have recalled several instances of this- the solutions given by Diophantus presupposed that the problem could always be expressed by one equation in one unknown, and thus transformed into a determinate problem. Sometimes he accomplishes this by arbitrarily attributing a value to one of the two unknowns. Some other time, he expresses the two unknown in terms of a third quantity, the unknowns of the (system of) equation. Gosselin's rethinking of this point is therefore particularly significant. On the one hand, as we have seen he carries over a series of procedure determinate problems from Diophantus at the end of Book III of ***De arte magna***, in which he gives a classification of equations. On the other hand he also develops

148 p. 61.

a theory of the indeterminate equations. This is the topic of Book 4, the last of ***De Arte magna***. Now, if Diophantus is helpful to *avoid* several unknowns, when it comes to establishing "systems" of indeterminate equations, the masters are Cardano, Stifel and Peletier. Yet, the theory of the second unknowns changes precisely in the ***De arte magna***. Gosselin's awareness of proceeding towards a transformation of this theory comes to light from his criticisms of earlier authors. In this regard he writes:

> In quarum regularum declaratione rationem a Luca, Stephano, Cardano, Buteone, aliisque communiter institutam non sequar, cum sit ipsa non fallax solum, sed & plerunque falsa.(***De Arte magna*** p.80)

We shall try to see how much Gosselin differs from his predecessors even when he decides to abandon Diophantus' precepts. This is not evident at first. It should be noted, for instance, that the mentioned passage is a paraphrase of a passage from the same Buteo,[149] cited by Gosselin.

What Gosselin makes decisively clear is the distinction Diophantus between determinate and indeterminate equations. In this sense, he choses some problems from Diophantus and puts them in the IV book. They are the one which can be better solved by using several unknowns. In this, he extends the theory already present in Peletier's book by applying it to Diopahtnus' problems. In addition, he transforms the relevant notation. While Peletier had taken over his notation directly from Stifel, thereby distinguishing the first unknown as "radix", Gosselin now revises it, already "denominating" the first unknown with the letter A.

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149 Buteo in fact wrote in the Logistica(p.189): "Superest aliud ratiocinandi genus, vulgo dictum Regula quantitatis, quadantenus simile quadraturae, una tamen positione non absolvitur, sed duabus, aut tribus, pluribusve, minimum autem duabus. In huius prosequutione formam a Luca, et Stephano, aliisque communiter positam ipse non sequar, cum sit omnium molestissima, captuque difficilis. Sit ergo propositu."

The use and transformation of Diophantus appears already in the first problem mentioned by Gosselin: "Partiamus 100 in duas eiusmodi partes ut prioris quadrans posterioris sextantem 20 superet." Gosselin proceeds in the following way:

> i.e. let us divide 100 into two parts such that one fourth of the first exceeds one sixth of the second of 20. We call the two parts 1A, 1B. thus 1A 1B will be equal to 100, and 1/4 A will be equal to 1/6 B P 20, and furthermore 1A equal 4/6 B P 80, and since 1A 1B are equal to 100, for 1A let us put 4/6B P 80, then 5/3 B[150] P 80 will be equal to 100, and after taking away the superfluous 5/3A is equal to 20, so is the aequation, we divide 20 by 5/3, so much will be 12 the first number A, hence the second, B will be 88.

This procedure can be expressed as follows:

We have * 1A + 1B = 100, and A and B are such that ** 1/4 A = 1/6 B + 20. Therefore, by **, 1A = 4/6 B + 80 , that we can replace in * and we get

5/3 B + 80 = 100, or 5/3 B = 20. Thus, B = 12, and by * A = 88 .

The problem is very simple, and the procedure clear: we choose as unknowns in the equations the unknowns in the statement of the problem. The data allow us to write two equations (what we would call a system), and we use what we would call the method of substitution.

In spite of the simplicity of the problem, we should be clear about the transformation (and improvement) made by Gosselin over the previous treatments. Gosselin has chosen as first problem something elementary and at the same time paradigmatic. At the end he writes: "Hoc aliter Diophantus ex Algebra sexto problemate libri primi." Aside for the interesting fact of writing Algebra in connection with Diophantus (but this is not surprising after reading Xylander), we should notice that Gosselin writes that Diophantus dealt with the

150 Here we have replaced B where Gosselin, or the typographer, making a trivial mistake, has A. However, since the conclusion is consistent with the mistake, thus Gosselin gets A = 12 and B = 88, we are lead to think that Gosselin made the mistake. Furthermore, if, as it is probable, Gosselin was consulting Diophantus, the mistake could have been suggested by Diophantus' order.

problem differently. In fact, Diophantus uses here, as in the previous problems, the "arithmòs", i.e, the name of a quantity which is not in the statement of the problem, not being the first nor the second number requested. Rather, both numbers can be expressed by the arithmòs, for he puts "one sixth of the second number" to be 1 N, so that the second number will be 6 N. By consequence, "one fourth of the first number" will be 1 N + 20, thus the first number will be 4 N + 80.

So, while the substitution occurs in both authors, the choice on unknowns in the equations corresponds of the unknowns of the problems only in Gosselin's text, and this is why the problem is solved "otherwise."

In fact, there are also other differences: first of all, Diophantus starts with a general problem: "Datum numerum in duos partiri, ut prioris par certa certam posterioris partem superet quanto iubebimur numero." This generality imposes a condition of possibility,[151] thus Diophantus continues:

> Hunc autem minorem oportet esse eo, qui ad dividendum nobis propositi numeri partem eam, quae alteri praestare debet, exprimit. Partiamur ergo 100 in duos numeros, ita ut prioris quadrans posterioris sextantem 20 unitatibus superet. Pono sextantem posterioris 1 N.[152]

This is how Diophantus arrives at establishing the arithmòs by starting from the "second" number. Thus, for Diophantus the problem was general, the solution was bound by the limiting conditions and solved on a particular case. By contrast, for Gosselin the problem was taken as particular, but the solution was general. The means used were somewhat disproportioned, in this case, but he was interested not in the actual solution for this simple

151 Also Xylander is concerned with the limiting condition, and, as Diopahntus, developsa a counterexample.

152 Xylander, pp. 15-16.

case, which could be solved in many ways, but in the way to solve it. which was general. Of course, this is not meant to be a statement about Diophantus, because he was also looking for general solutions, in another context, but as a point about Gosselin, who makes use of Diophantus. Gosselin shows here to neglect the context of Diophantus' theory, and to be attentive only to the possibility of using Diophantus' cases as paradigms of the main categories of problems.

Let us now go back to Gosselin's book as such. Gosselin's treatment of the equations in several unknowns is called, in his book, "the absolute (or simple) quantity" and "the surd quantity." In this, once again. Gosselin imitates Cardano's use of the word "quantity" for the second unknown. Less clear is the distinction between the two. Bosmans has wondered what this distinction could mean. In fact, in our view, there is no distinciton between the problems studied in the first group (problems under the heading *De quantitate absoluta*) an the problems studied in the second group (under the heading *De quantitate surda*). This distinction does not correspond to the tradition: Pacioli called *surda* simply the second unknown. Cardano does not use the word *surda* in this sense. Peletier did not distinguish between two methods for the second unknown, and in general the word *surda* meant irrational, as it is its common meaning. Borrel calls the section on second unknowns ***De regula quantitatis***. Given that Gosselin himself does not give a definition of the surd quantity, we must understand it from the examples. Let us take an example similart to the one we have seen developed by Peletier. "To find three numbers such that the first plus 8 is 1/3 of the sum of the others, the second plus 8 is 3/5 of the sum of the others, the third plus 8 is equal to the sum of the others. Let the first number be 1L; if we add 8, then 1L + 8 is one third of the others, and their sum is 3L + 24. Let the second number be 1q, so 1q + 8 will be

3/5 of the others, and their sum will be 5/3q + 40/3. Since the first is 1L, let us take it, there will remain 5/3q + 40/3 - 1L, i.e. the third number. Let us now add to it 1q, and we shall get the sum of the second and the third, i.e. 8/3q + 40/3 - 1L. But we had already expressed this sum, so Gosselin writes the equation 8/3q + 40/3 - 1L = 3L + 24, we get 8/3q = 4L + 32/3. Thus we get the expression for 1q, 1q = 3/2L + 4. We can now take it from the sum of the second and the third, ie.e. we write: 3L + 24- 3/2L -4, and we get 3/2L + 20, which will be the third number. Because of the hypothesis on the third number, this will be equal to the other two, so 3/2L + 20 + 8 = 5/2L + 4. Hence 3/2L - 5/2L = -28 +4, so l = 24. From this we can obtain q = 3/2L + 4 = 40, and finally the last number, 56.

Now we can ask ourselves what characterized this procedure (which is common to the other three examples given in the text), and then why Gosselin considered it separate from the previous one.

At first, we notice that Gosselin does not make use of the previous notation, but of a new notation which we can recognize as the latus L and the quantity q. Secondly, there are only two unknowns, and the third is expressed in terms of the other two. However, and this is a difference with respect to Cardano,[153] the two unknowns of the equations are also the unknowns of the problem. Finally, the solution is obtained only by substitution, there is no use of the method of sum and subtraction of the equations, because the coefficients are different.

In conclusion, we might still wonder, as Bosmans does, why Gosselin does not apply the procedure of the "absolute quantity" also to this and to the other problems of the "surd quantity", as we would do, creating a system of three equations. He *could* have done it. The

153 I am referring to the chapter "On the second unknown quantity, multiplied", i.e. chapter X of the ***Ars Magna***.

answer seems to be that Gosselin conceived of this only when addition and subtraction are possible. Furthermore, we might add that Gosselin took this distinction from Nunez.[154] Nunez, who published his work only in 1567, had finished it about twenty years earlier. So, he deals with the topic in a much less explicit way, while Gosselin shows a much greater flexibility. However, Gosselin had the previous definitions in mind, and he detached himself form his predecessors only to a certain extent, i.e. only when he was authorized by another authority, that of Diophantus.

A concluding remark on Gosselin's notation: it is true that this innovation originates with Borrel, but Gosselin uses it with a new skill that permits him to more easily solve the same problems proposed by Borrel.[155] It seems reasonable to think that Viète took this symbol as a point of departure to arrive at his A,E.[156] Gosselin could also be a source for the notation used by Descartes, who, in the ***Regulae*** proposes to designate the known terms with lower-case letters and the unknowns with capitals (Regula XVI).[157] Certainly, the

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154 See ***Libro de algebra***, p.224v.

155 One should also remember that priority on this topic has not been established. In particular, Stifel has listed as a possible algebraic notation the "Cossische Progression" 1A, 1AA, 1AAA, ... and similarly with the other letters of the alphabet. This is what we find in early seventeenth-century algebra, but what seems relevant is that Stifel does not use this notation. (See F. Cajori, ***A History of Mathematical notation***, Open Court, Chicago 1929, vol. I, p. 144). What we have seen is one element in the French algebraic tradition.

156 We read in the ***In artem analyticen Isagoge***: "Quod opus, ut arte aliqua juvetur, symbolo constanti & perpetuo ac bene conspicuo datae magnitudines ab incertis quaesitiis distinguantur, utpote magnitudines quaesitas elemento A aliave litera vocali, E,I,O,V,Y, datas elementis B, G, D, aliisve consonis designando."(François Viète, ***Opera Mathematica, recognita Francisci a Schooten***, Vorwort und Register von J.E.Hofmann, Olms, Hildesheim 197O.

157 Cfr. René Descartes, ***Regulae ad directionem ingenii***, texte critique établi par Giovanni Crapulli, La Haye 1966, p. 72: "Quidquid ergo ut unum ad difficultatis solutionem erit spectandum, per unicam notam designabimus, quae fingi potest ad libitum. Sed, facilitatis causa, utemur characteribus, a, b, c, &c. ad magnitudines jam cognitas, & A, B, C, &c. ad incognitas exprimendas;."

Borrel-Gosselin notation is particularly significant insofar as it indicates that these authors had codified a strategy that would then be adopted by Viète and made explicit by Descartes, that is, to introduce as many symbols for unknowns as there are unknowns in the problem.

In conclusion, Stifel transmitted Cardano's doctrine on the second unknowns to the Parisians, while acknowledging Cardano's authority on the subject. Jacques Peletier and Guillaume Gosselin imported also Cardano's ***De arte magna***, and considered these developments worthy of belonging to their "agenda", with the result that they came to be a mark of the French algebraic tradition.

### *5. Dimension and Degree*

Another element in the algebra of Cardano constituted a point of reference for the manuals that came immediately thereafter. It concerns the meaning to be attributed to powers of unknowns beyond the third degree, and as a consequence, whether it is fitting to take an interest in problems and equations beyond the third degree. The question is not trivial, as some historians at the beginning of the twentieth century seemed to think.[158] Without the lens of the retrospective historian, it is far from obvious why an author would study equations of a more complex type than the problems for which he was seeking solutions. In fact, new and different problems did eventually arise somewhat later to give importance to the equations of higher degrees. We will now consider this topic, which also includes the problem of the transmission of the solution of the cubics.

Cardano writes at the beginning of the ***Ars magna***:

Et quanquam longus sermo de his haberi posset, at longa capitulorum series

158 See in particular H. Bosmans, "Le 'De Arte Magna' de Guillaume Gosselin, ***Bibliotheca Mathematica***, Vol. VII (1906-7), p. 44-66. He deals with this aspect on pages 55-56.

> subiungi, finem tamen exquisitae considerationi in cubo faciemus, caetera, etiam si generaliter quasi tamen per transennam tractantes, namque cum positio lineam, quadratum superficiem, cubus corpus solidum referat, nae utique stultum fuerit, nos ultra progredi, quo naturae non licet. Itaque satis perfecte docuisse videbitur, qui omnia, quae usque ad cubum sunt, tradiderit, reliqua quae adiicimus, quasi coacti aut incitati, non ultra tradimus. (***Opera Omnia***, p. 222)

This last statement, therefore, had not only the purpose of recalling the geometric meaning of the Cossic numbers, but also of sketching a criterion of classification. Around that time, it was becoming common in algebra manuals to criticize previous authors for their multiplication of the canons, which is to say of the configuration of equations by means of which one provided a solution. Two theoretical needs came to be combined: first, to expand the procedures for the reduction to the known equations (i.e. the canons) and, second, to give more general rules for irreducible canons. Thus, in the texts of these authors, the theme of permissible dimensions of Cossic numbers came to accompany that of the reduction of canons to a reasonable number. In particular, this suggested to some authors that it would be inopportune to deal with cubic equations, since that would require the addition of canons still contested in their generality. Cardano, by contrast, preferred to give an account of the cubics, without regard for the further multiplication of canons. However, we have already mentioned above that the solutions published by Cardano in the ***Ars Magna***, despite having provoked a dispute over priority, were not immediately adopted in the manuals of the period.

Stifel had resolved the matter by putting in the foreground the all-encompassing rule *amasias*, which we interpret as the formula for the resolution of second degree equations, presenting it as a solution applicable to any kind of equation. It was in fact general, working regardless of distinctions of form, but only for the second degree. It seems as if his success with the second degree led Stifel to avoid the problem of cubic equations.

Peletier, who relies on Stifel's treatment, followed him on this point as well. Nevertheless, he was explicit in declaring the defects of Stifel's text. This is the way he summarizes his criticism of Stifel and Cardano and their algebra manuals:

> Retournant donc à nos Ecritteurs, je dirai, que de ceux que j'ai vus, l'un [Stifel] l'a traitée imparfaitement. Et si s'est vanté qu'il n'était possible de trouver d'autre généralité que celle par lui balhée: combien que Cardan l'ait augmentée de règles plus singulières et nouvelles, qu'il ne les estimait impossibles. De celui ci je dirai, qu'il l'a enrichie de belles inventions, avec demonstrations laborieusement cherchées, mais un peu confusément et très obscurement.(...) En somme, je dirai de tous ensembles, qu'ils ont eu peu d'égard à la méthode et ordonnance. (***L'algèbre***, p.3)

Concretely, then, Peletier followed the classification of Stifel, but not his reduction of every problem of classification to the "règle générale de l'algèbre." As for the treatment of third degree equations, Peletier saves this topic for the end of the second part, which deals with irrational quantities. He proposes to compare his solution of the cubic root under consideration with that given by Cardano for the same example. The fact that one finds here an explicit reference to chapter XI of ***De Arte magna*** shows all the more clearly the significance of Cardano for Peletier. We should also mention that on this very page he reveals his intention to write a third book of ***L'Algèbre***, in which he will add some surprising things regarding, one supposes, cubic quantities.[159]

As for Gosselin, he not only treats briefly the operative aspect of the question, but goes on in his chapter entitled *De aequatione tertia* to discuss in broad terms the inappropriateness of integrating into a treatise equations of this kind. After having reminded his reader that this type of equation had exhausted fruitlessly many of the great minds of mathematics, amongst which he counts Archimedes, he writes:

> multi quoque in hac perquirenda aequatione plurimum olei consumpserunt,

159 ***L'Algèbre***, p.194.

> inter quos Cardanus opus hoc videtur confecisse, sed quid ipse confecerit cum omnibus fere in exemplis ratio sua non constet? vera quidem illa sed saepius incognita, praeterea suae inventionis viam difficultas praecipua comitatur, quo fit ut hoc quod ille vocat egregium inventum non magni faciam, cum Binomiorum latera Cubica difficile difficilique labore inquirantur, inquisita saepius non cognoscantur, at hic universam viam quaerimus.(***De Arte magna***, p.71v)[160]

Hence, the search for solutions is not satisfactory from the systematic point of view. Cardano took it upon himself to demonstrate too many things, and in this way he demonstrated none of them in an adequate manner.

Gosselin's position here, with respect to third degree equations, is particularly indicative inasmuch as he already had a highly developed system of classification, following in the footsteps of Cardano (in the ***Ars magna***) and Diophantus. One could say, then, that having by this time achieved a definitive systematization of first and second degree equations Gosselin declines to treat the canons of third degree equations because he is not prepared to lay them out with comparable clarity. In fact, he concludes:

> Expectemus ergo donec summus ille Matheseos author aliquid horum generaverit in animis hominum. Interea pro nostro ipsi modulo de his nonnihil differamus, doceamusque aequationes omnes quae latuerunt hucusque hoc invento problemate etiam ipsas inveniri.Problema vero est eiusmodi. Vestigare lateris valorem cum Cubus et Quadrata sunt aequalia numero, vel contra variatis quantitatibus.(p.72)

Gosselin showed in this way that he had studied the problem and planned a new systematization. Cardano, by contrast, reduced this type of equation to the case including the cube, the first power, and the number, which was solvable by means of the famous rule.[161] From another point of view, if one considers that already in the ***Ars Magna*** Cardano

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160 This passage continues: "Neque propterea magnipendo particulares omnes ad hanc aequationem regulas quas fere omnes afferunt, verum praecipue Cardanus, falsas etiam saepius multum partialibus delectatus."

161 Chapter XI of the ***Ars Magna***, "De Cubo et Rebus aequalibus Numero." To give a sense of

had included even the results of Ludovico Ferrari on fourth degree equations, Gosselin does not appear to be very much ahead of his time.

He had, however, already in the second section of ***De arte magna***, defended the thesis, by that time taken for granted, supporting the reduction of canons in the nascent theory of equations:

> Non desunt qui portentosa canonum multitudine totam hanc artem contaminarunt, quae ipsi capitula appellarent: alii duos solum canones nec plures admiserunt, quasi vero nostra algebra tam arctis Laterum et Quadratorum finibus concludatur, at certe praestantissimi Arithmetici Lucas Pacciolus et Leonardus Pisanus rem ipsam paulo prudentius meditati neque duos neque infinitos asseruerunt posse dari canones, sed quemadmodum omnis quantitas continua vel est linea, vel superficies, vel corpus (locus nanque a superficie non distinguitur) sic artem hanc dixerunt circa haec tria versari, vel circa ea quae his respondeant in numeris, ut sunt Latus, Q. C. quo circa tres voluerunt commode posse fieri canones, nimirum unum simplicem et linearem, secundum planum qui de Quadratis agit, tertium denique solidum qui de Cubis quam ipsi tueantur necnon sententiam non plane constat, an intelligant de tribus canonibus, ad Quadrata, vel de tribus hoc est ad Latera Quadrata et Cubos, ut ut sit nostra haec est sententia quam postea iuvante Deo demonstrabimus. (p.53)

Thus it is that all the elements of the question of dimension and degree resurface in Gosselin, together with his awareness of the incongruences and the ambiguity of some earlier formulations. On the one hand, it is evident that Pacioli had been too restrictive, while on the other, his discourse on the dimensions, later replayed by Cardano, seems actually to refer both to dimensions and to canons of the second degree.

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the mathematics involved, I copy here the text of Cardano's first example in Witmer's translation (The Great Art or the Rules of Algebra by Girolamo Cardano, Translated and edited by T.R.Witmer, M.I.T. Press, 1968): "For example, $x^2 + 6x = 2O$. Cube 2, one-third of 6, making 8; square 1O, one half of the constant; 1OO results. Add 1OO and 8, making 1O8, the square root of which is √1O8. This you will duplicate: to one add 1O, one-half the constant, and from the other subtract the same. Thus you will obtain the binomium √1O8 + 10 and its apotome √1O8 - 1O. Take the cube roots of these. Subtract the cube root of the apotome from that of the binomium and you will have the value of x: $\sqrt[3]{}$√1O8 + 1O - $\sqrt[3]{}$√1O8 - 1O." In order to follow the rule, one should remember that binomium and apotome are the two words for our notion of binomial, for in our notion we do not distinguish the sign.

However, it is clear as well that at this point Gosselin reinterprets a *topos* found in earlier authors in order to introduce his classification. The textual reference to Cardano was a way for him to call on the authority of the tradition, while his actual interlocutor was Jean Borrel, with whom he was for once in agreement against the practice, if not against the thesis, of Cardano. In fact, Borrel in his ***Logistica***, introducing his version of the second unknown with the name *regula quantitatis*, dedicates an entire paragraph to the question *An in quadratura canones compositi plures tribus commode fieri possint*. At this point, Borrel takes up again the thesis and the argument of Pacioli, and expresses his disagreement with Cardano, not only insofar as he multiplied the canons but also because without having directly criticized Pacioli's criterion he had altered it. Pacioli had allowed only six canons, three simple (in which each member was constituted by only one term) and three composite (in which a member was constituted by two terms). The geometric interpretation, then, was limited to line and plane. Cardano, as we have seen, takes over the tenor of the passage from Pacioli, but extends its meaning to solid bodies, or the third degree. As we might expect, Borrel,[162] who like Pacioli mentions the tenth book of Euclid,[163] takes him to task on this point.

We have followed this dispute from its end in the mid-sixteenth century back to its

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162 "Disputans Lucas de compositis canonibus asserit, non plures tribus fieri posse. Cui sententiae Cardanus in opere (...) non verbis solum, sed re ipsa valde repugnat. Totum enim librum prodigiosa canonum multitudine constipavit, capitulorum nomine vocans, atque distinguens, in primitiva, derivativa, imperfecta, particularia, maiora, singularia, et multis modis aliter (...)." (Logistica, p.187). According to Borrel, for the purposes of logic, the six canons allowed by Pacioli are enough. "Novos autem, quales fecit Cardanus, et alii, quos citat, hoc est vanos, imperfectos, implicitos, particulares, non dico multos, sed infinitos posse constitui."

163 Prop.115. According to Heath's translation: "From a medial straight line there arise irrational straight lines infinite in number, and none of them is the same as any of the preceding."

beginning, in 1494, with the publication of Pacioli's ***Summa***. In brief, then, let us review the main elements. Cardano based his work on that of Pacioli, but selectively, and only in order to extend Pacioli's considerations to the third dimension. Borrel, less critical, takes over whole-heartedly the thesis of Pacioli, according to which there are only two geometric figures or dimensions useful in algebra, and six canons. Finally Gosselin, after having discussed in a masterful way the various theses, opted for three dimensions, but welcomed Borrel's criticism of Cardano's ability to make demonstrations, and his lack of a systematic sensibility.

It is precisely the theme of mathematical demonstration which allows us to arrange these various positions in perspective. Cardano, as he himself explained in the manuscript published as ***Hexaereton mathematicorum*** (composed in 1572), made no claim to give, in algebra, demonstrations in the "true and proper" sense.

> "in ea quorum inventio ex arte magna habetur, demonstratio vero adiicitur ut non tantum sciamus (est enim scientia quae per demonstrationem habetur) sed ut unum in aliud mutare discamus, idque appellatur, si purum fuerit, restitutio."(***Opera Omnia*** p.446)

In other words, in algebra we should not look for demonstrations, but for "reconstructions."

Borrel and Gosselin place themselves in the opposing camp, both of them being preoccupied with giving a scientific status to algebra, rather than allowing it to be simply an art propedeutic to science, as Cardano seemed to indicate.[164] We shall not recall here Gosselin's theses on this topic.[165] Instead, I will cite the words of Borrel, in which he attributed a precise goal to Pacioli's choice to reduce the canons to the first two dimensions:

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164 See the rest of the prooemium.

165 Particularly interesting for this question, along with the introduction to the ***De Arte Magna***, is ***De ratione discendae docendaeque mathematices***, as we shall see in the next chapter.

> Intelligebat etiam regulas huiusmodi, non tam ad communis usus necessitatem, quam ad meditationem subtilitatis inventas. Quae cum circa disciplinas pateat in immensum, nisi certis legibus intramodum exerceatur, magis est onerosa quam utilis, nec tam excitat, quam obruit ingenium.(p.187)

We have then, on one side, Cardano -- algebra as an art, rigorously geometrical interpretations of the degree of equations, and therefore, limits on dimensions, and unlimited and always unconstrained heuristic use of dimensional symbols in the manipulation of problems or functions.

On the other side we have Borrel and Gosselin -- algebra as science, and the progressive assimilation of elements into a system meant to be theoretical and which will find its definitive codification in Viète's symbolic algebra and theory of equations.

* * * * *

## Appendix 1

Notations for the powers of the first unknown

[read down each column to compare authors]

**<u>Contemporary notation</u>**

| | | | | | |
|---|---|---|---|---|---|
| 1 | x | $x^2$ | $x^3$ | $x^4$ | |

Cardano, *Practica Arithmeticae*:

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 co. | 1 ce. | 1 cu. | ce.ce. | etc. |
| | (una cosa) | (un censo) | (un cubo) | (censi censo) | |

Stifel, *Arithmetica integra*:

| | | | | |
|---|---|---|---|---|
| 1 | | | | |
| | (radix) | (census) | (cubus) | (quadratoquadratus) |

Cardano, *Ars Magna*:

| | | | |
|---|---|---|---|
| 1 | res ignota<br>[or] positio | quad. | cub. |

Peletier, *L'Algèbre*:

| | | | | |
|---|---|---|---|---|
| 1 | | | | |
| | (racine) | (çanse) | (cube) | (çansiçanse) |

Gosselin, *De Arte Magna*:

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | L. | Q. | C. | QQ. | RP. | QC. |
| | (latus) | (Quadratus) | (Cubus) | (Quadratoquadratus) | (relatus primus) | |

Notations for the second unknowns

<u>Contemporary notation</u>

| | | | | |
|---|---|---|---|---|
| x | y | z | $y^2$ | $y^3$ |

Cardano, *Practica Arithmeticae*

| | | | | |
|---|---|---|---|---|
| 1 co. | 1 quan. | | | |

Stifel

| | | | | |
|---|---|---|---|---|
| 1 | 1 A | 1 B | 1 A | 1 A |

Cardano, *Ars Magna*

| | | | | |
|---|---|---|---|---|
| 1 pos. | 1. quan. | | | |

Peletier

| | | | | |
|---|---|---|---|---|
| 1 | 1 A | 1 B | 1 A | 1 A |

Gosselin

| | | | | |
|---|---|---|---|---|
| 1 A | 1 B | 1 C | 1BQ | 1BC |

Appendix 2

From M. Stifel, ***Arithmetica Integra*** p.154

Exemplum Capitis huius Tertium
et est Hieron. Card. 28 Capitis 66

Sunt tre numeri continue proportionales, cum quorum quolibet divido 25 & invenio quotientes trium harum divisionum simul sumptos, facere eam summam, quam facit multiplicatio eorum divisorum, inter se, quamque additio eorundem divisorum ad se facit.

Quaestio est qui sint numeri illi. Non autem hic poteris recipere.

1 . 1 . 1

Nam tunc necessario fieri significaretur, secundum numerum esse quadratum primi, Id quod fieri necesse est solummodum progressio incipit ab unitate, &c.

Recipe ergo, 1 , 1A, 1B. Tunc quotientes sic stant

| | | |
|---|---|---|
| 25 | 25 | 25 |
| 1 | 1A | 1B |

Hic observa, quod medius minutiae denominator aequatur ipsi minutiae mediae, eo quod denominatores sint ad invicem proportionales, sub aequalibus numeratoribus. Itaque 1A aequatur . Facit igitur 1A .25. Et ideo 1A facit 5. Unde sic stant numeri exempli a modo.

1 5. 1B.

Et cum numeri ad invicem sint proportionales, sequitur multiplicationem extremorum inter se, aequari producto multiplicationis medii in se, scilicet 1 B aequatur 25.

Quia vero in pronunciatione exempli habes, quod multiplicatio eorum numerorum inter se, debeat facere summam aequalem ei quae fit ex additione numerorum illorum inter se, ex 1 vero in 1B fiant 25. multiplicanda per medium, id est per 5. sequitur summa aggregationis facere. 125.

Itaque pronunciatio exempli praesentis iam versa est in pronunciationem hanc.

Dividitur numerus 125. in tres partes adinvicem proportionales, quorum medius est 5. Quaeritur ergo quanti sunt extremi.

Sic stant numeri iuxtam pronunciationem hanc

1 5. 120-1

Nam 125-1 -5. facit 1B. Erit ergo aequatio inter 25. & 12 - 1 . Cum sint numeri proportionales. Itaque 1 aequatur 120 -25. facit 1 . 60 - 3575, et est radix aequationis minor. Nam maior radix facit 60 + 3575.

Sic ergo stat inventa progressio. 60 - 3575.5.60 + 3575.

Nisi autem utereris hac industria reducendi pronunciationem priorum ad posteriorum pronunciationem, negocium tibi nasceretur cum 1x AB aequata

$$25AB + \frac{25\ B}{1\ AB} + 25\ A$$

from Girolamo Cardano, ***Practica Arithmeticae***, cap. 66, par.28

***Opera Omnia***, vol. IV, p.144

Habui tre quantitates continue proportionales et per singulam illarum divisi 25. et proventus tres aggregati fuerunt tantum quantum illae tres quantitates et similiter tantum fuit illud quod fit ex prima in secundam et producto ducto in tertiam quaeruntur quantitates illae: tunc tu scis quod illud quod fit ex prima in secundam et producto in tertiam est aequale cubo secundae quantitatis per nonagesimam regulam quadragesimi secundi capituli, igitur cum tale productum aequetur dictis tribus quantitatibus erunt dictae tres quantitates iunctae cubus secundae quantitatis et quia tamen aliqua quantitatis dividitur per tres quantitates continue proportionales ita quod provenientia iuncta sint aequalia dividentibus tunc secunda ex illis quantitatibus est numeri dividendi quadrata: per regulam nonagesimam primam quadragesimi secundi capituli igitur secunda quantitas est quadrata 25. hoc est 5. & ipsa erat cuba aggregati igitur aggregatum est 125. igitur dempta secunda quantitate remanent reliquae duae 120 & quia ex prima in tertiam tantum fit quantum ex secunda in se per regulam 104. capituli 42. & ex secunda in se fit 25. igitur dividemus 120. in duas partes quarum una in aliam ducta faciat 25. eritque per capitulum 49. una 60. p. 3575. alia 60. m. 3575. & media illarum fuit 5. & ita soluta est. Frater autem Lucas posuit eam & soluit cum magna difficultate & pluribus operationibus superfluis.

* * * * *

# **Chapter Four**

## **The *De ratione discendae docendaeque mathematices* of Guillaume Gosselin**

***Part A. Introduction: Rhetoric and patronage***

In this chapter we examine a text which, although it has been printed, cannot be considered as published. It exists, to my knowledge, only in a gift edition of one parchment copy. This is kept at the Bibliothèque Nationale in Paris, Réserve des livres rares et précieux.

We have already mentioned, in the first chapter, the social roles of the people cited in the text: they are *maîtres de requêtes* or member of the *Conseil du Roi*. Gosselin addresses himself to these high magistrates to obtain patronage, and it seems that at least in one case Gosselin could count on a specific competence in the subject, for he mentions, among others, François Viète.

The *genre* of this work is that of a *praelectio*, i.e. of a sort of "syllabus" for a course, which was often presented as such in a lecture. In fact, this is described as *repetita praelectio*.

The structure of the text is similar to that of the *genre* of *De quantitatibus*, common in the sixteenth century after the rediscovery of Proclus' ***Commentary to the first book of Euclid***, which was published at first in 1533 and made famous by a work which could not have escaped Gosselin's attention, Ramus' ***Scholae mathematicae***.

Thus, Gosselin discusses a series of classical problems in the philosophy of mathematics. This discussion includes a definition of mathematics and of its parts (what is mathematics, the continuous and the discrete), a classification of mathematical disciplines, the parts of geometry, the kinds of propositions (problems and theorems), axioms and postulates and their nature. Here Gosselin does not enter into a discussion on common principles, as we could expect from a Ramist, but instead lists the main definitions of geometry. Then he goes on to arithmetic. But he dismisses speculative arithmetic insofar as (he writes) its treatment does not differ from that of geometry, except that instead of lines one deals with numbers. So he enters directly into the matter of practical arithmetic, which he calls, in a Ramist fashion, acting (*agens*) as opposed to practical.

Within "acting" arithmetic, the distinction is between rough arithmetic (*rudior*) and subtle arithmetic (*subtilior*). Subtle arithmetic is, of course, algebra. This means that algebra is included in mathematics in the strict sense, i.e. arithmetic and geometry, and this implies some adjustments in the theory. In particular, it appears that Gosselin gives priority to problems over theorems and, at the same time, includes proofs in algebra. In general he tries to present algebra in a manner that imitates the way of presenting geometry. For instance, while the problems still open in geometry are the three classical ones (quadrature of the circle, duplication of the cube, and trisection of the angle), the problem still open in arithmetic is the solution of all third degree equations. While excluding the solutions for the third degree equations, Gosselin includes the results of his study of Diophantus, in particular the various kinds of equations defined in the ***Arithmetica***.

Now, to the text.

* * * * *

# Gulielmi Gosselini Cadomensis Issaei

# De ratione discendae docendaeque mathematices
# repetita praelectio

Ad Ioannem Chandonium et Carolum Bocherium
supplicum libellorum in regia Magistros

M. D. LXXXIII.

//

Amplissimis viris Ioanni Chandonio et Carolo Bocherio supplicum libellorum in regia Magistris

Guliel. Gossel. S. D.

*Lapsi sunt in eo recentiores prope omnes (Viri clarissimi) quod cum Mathematicen explicare vellent, huius involucro partium sunt implicati, neque vero canones percensuerunt erroris vacuos, sed alterius exempli regulis, angustissimis nempe finibus, augustissimam scientiam terminarunt: feceruntque adeo, ut absolutum nihil Mathematicae candidatus ex ipsorum scripsis eduxerit. Enimvero, ut, quod res est, dicam, iam me inter Mathematicos ordinem adsecutum super ipsorum //v libris varius labor exercuit, quorum opera ab ipso praelo si occidissent, fausto rursum surgeret omine*

Millibus e multis hominum consultus Apollo[166]

*Hoc est (ut Platonicus interpretor) non Deliorum instar imprudentium fugeremus Mathematicen, sed quoad eius fieri posset, omnes in universum totas animi facultates huic adiungeremus, at quasi difficilis obstitit aditus, obscurumque caliginosis limen oculis studiosum absterruit: queis ego consultus optime, ea dederam in numeris, quae facilitate reliquis praeirent, doctrina nullis cederent: itaque & candidatus & provectior reportabant, uterque quod posceret, herbamque ille auctor porrigebat nemini: unum hoc in homine non vulgari Mathematico desiderari videbatur, ut quamprimum facile quoddam rudimentum expedirem, quod magistra placuit adnuente experientia. Cum ergo negociosa inter ocia legerem vix quiescens, forteque in docendae discendaeque Mathematicae rude quoddam figmentum incidissem, opusque examinassem adcuratius, visum est primo extimoque obtutu non indignum lu//3 cis, quippe duce illo*

166 "Vir bonus et sapiens, qualem vix repperit unum / Millibus e cunctis hominum consultus Apollo,/ Iudex ipse sui totum se explorat ad unguem." ***Appendix Vergiliana. De Institutione viri boni***, l. 1,3. Thanks to Dr. Deborah Lyons for assistance.

*Mathematicis sese monstrantibus: tantum urgebat denique & efflagitabat candidus erga studiosos amor, quantum habenas non potui comprimere: accedebat quod vestri utriusque auspiciis iturum esset in publicum, dicturumque vobis: hoc vos accipite primum aeternumque mei erga vos officii argumentum, nosque amate & suum auctorem, quanquam ille, ut Horatius loquitur,*

Fungitur vice cotis, acutum

Reddere quae ferrum valet, exors ipsa secandi.[167]

*Valete//*

---

167 "Nil tanti est. Ergo fungar vice cotis, acutum reddere quae ferrum valet, exsors ipsa secandi." Horace, ***Ars poetica***, l.304, 5.

De ratione discendae docendaeque mathematices

repetita praelectio.

## Quod doctrina, disciplina & scientia unum idemque sint

### *Cap I Dist I*

Si nulla esset doctrina, nec aliqua esset disciplina: quod si nulla disciplina, aeternum tacerent literae: quare submota doctrina, nullaque existente disciplina, scientia removetur. Nanque cum intellectus in singularia compositus universam quandam notionem ex mutua sensilium concordia dignoscit, & seipsum docet, & discit ex seipso: quo de entium genere sensus doctrinam, quam nos peregrino nomine dicimus adprehensionem, secludimus, ut doctrinae disciplinaeque appellatio sub uno //4 intellectili concludatur, atque iccirco cum scientia subsistem consecutionem habeat.

### *Cuius sit doctrinae generis Mathematica*

### *Dist II*

Triplex est in homine functio, sensus, intellectus, &, quae particeps est utriusque, phantasia: sensus est individui, praesentis: phantasia, individui, absentis: intellectus, tum praesentium, tum absentium, individuorum, prout inter se componuntur, universa quaedam affectio: ac quidem sensus est adprehensio, phantasiae imaginatio, unus sub disciplinam cadit intellectus: missis ergo reliquis, quot sint disciplinarum genera perscrutemur: iam quicquid intelligitur, vel demonstratio, vel fides confitetur, vel confirmat auctoritas: unde planum est omnem disciplinam, vel esse philosophiam, vel sanctam Theologiam, vel historiam: duabus praetermissis descendamus ad eam, //v quam philosophiam appello, scientiam. Haec autem quintuplicis est differentiae: aut enim disserendi subtilitatem docet, vel quaestiones naturae obscuras tractat, vel vitam atque mores definit, vel agit de ente ut ens est, aut denique quantitatum affectiones monstrat, & haec Mathematica dicitur, qua de nostra instituitur disputatio.

### *Cuius sit scientiae gradus Mathematica*

### *Dist III*

Omnem doctrinam, omnemque disciplinam antecedente fieri principiorum notione constat, principia scientiae non aliter cognosci quam supponi, itaque demonstrationem positione nasci. Huiusque effectum purius esse eo quod a definitione progreditur, quippe in infinitum demonstrari finitiones possunt, nanque & finitio finitionem habet: principii tamen non est principium. Cum vero duplicis scientiae demonstratior illa sit quae principiis //5 nititur quam quae finitione, quo certe manifestiora sunt principia, scientiam eam esse demonstratiorem pro confesso habebitur:

queis ita consistentibus, ut quintuplicis philosophiae pars Mathematica, notioribus insistat incedatque principiis, merito prae ceteris, disciplinae, scientiae, doctrinaeque nomen sibi vendicat: nec obstat quod philosophorum nominalium iniquum agmen uno ore Metaphysicam scientiam esse omnium certissimam pronuntiat, ut quae nec re, nec ratione Physicae inhaereat materiei: Mathematica quanquam ratione, re tamen non separet, sed horum error est in conspicuo, quandoquidem scientiarum, ut sic loquar, intensio ex obiecto, non ex subiecto deprehenditur: ideoque tametsi Metaphysica neutro modo materiei subsit, Mathematice subiaceat altero, puta triangulus ligno: tamen non hoc aut illud triangulum adsumentes, sed generale quoddam obiicentes ex hoc aut illo subiectis, secundum certissima prin //v cipia concludimus triangulum omne treis habere angulos duobus aequos rectis.

### *Quod sit obiectum Mathematicae*

### *Dist IIII*

Sunt qui persuasum habent figuram qualitatis speciem obiectum esse Mathematices, in quo inepte nimium hallucinantur, cum Aristotelem vestigantes ab ipso declinarint longissime: est enim philosopho figura duplex, quoniam ipsa considerari bifariam potest, vel per triplicem continuae quantitatis affectionem, vel per terminorum distinctionem: ac quidem altera ratione qualitas est, priore quantitas: quia tamen indistincte figuram ut figura est fecerunt obiectum Mathematices, falsum quoque &

inane Mathematices obiectum constituerunt: etiam vero nobis adnuentibus quod figura ut qualitas sit quantitas, non continuo dicetur obiectum, sed obiecti, ut figura //6 sit obiecti Mathematicae: res in aperto sit, deturque numerus: nec est potior ratio cur figura sit obiectum, quam numerus, namque ut se habet figura ad Geometriam, sic numerus ad Arithmeticam, quarum utraque partium aequa pars est Mathematicae: itaque refutata superiori, Philosophorum, Mathematicorumque; omnium haec esto sententia, quantitatem obiectum esse Mathematices, ne tam arctis figurae limitibus Mathematica terminetur.

### ***Quod sint Mathematice species***

### ***Dist V***

Certum est distinctas species ex obiecto nasci, ac Mathematicae quidem obiectum esse quantitatem illam quae secundum supremum ens creat: hoc vero ens duae dividunt differentiae, coniunctum & disiunctum, quae duo rursum subalterna quaedam genera faciunt: unum, quod longum, latum, altumque comprehendit: alte //v rum sub quo numerus concluditur, prius Geometria, posterius dicitur Arithmetica: neque hic plura Mathematicae genera possunt excogitari, quoniam quicquid quantum est, vel quantum est lineate, vel quantum numerice: simplex lineatum, ut circulus in planis, vel in solidis Sphaera: compositum, ut Quadratum in planis, in sphaericis corporibusque Cubus: numerice quantum, ut ternarius in absolutis, in partibus semissis: ac ideo duo Mathematicae genera, duasve species secundum obiecti differentias constituimus, Geometriam & Arithmeticam. Geometria continuum, ut continuum est, tractat: nil est enim motus, loci, temporisve sub nomine continui, cum motus per universa decem entia diffundatur, nec possit non pertinere ad illud cui adplicatur: locus ab essentia superficiei non discrepet, cum suae sit pars finitionis: tempus denique praeter instans nullum sit, quod saltem indirecto revocaretur ad continuum, quemadmodum pun //7 ctum, monas et mutatio reducuntur. Arithmetica numerum, ut numerus est, considerat, nanque orationem sub devincto non collocamus, ut tollamus infinitum, et numeratum seiungamus a numero. Iam utriusque multae sunt species, prout continuum & discretum varias sortiuntur affectiones: continuum multiplex est: terrestre, quod Geodesiam constituit: coeleste, quod Astronomiam: aerium aqueumque duplex, vel ponderis, ut Pneumaticum & Automatopijticum,[168] vel visus, ut Optice: vel denique promiscuum, ut Mechanice: disiunctum quoque multiplex est, propter triplicem numeri proportionem: vel enim aequorum excessuum in terminis habetur ratio, quod rudiorum facit Arithmeticam, vel multiplicium, quod subtiliorem, vel commixtorum ex aequis & multiplicibus, quod consonantes. Iterum utraque quantitatis species bipertito dividitur, in agentem & cognoscentem: haec leges facit, regulasque condit: illa cognoscenti in //v nixa quam volebat actionem consequitur: prioris sunt Problemata: Theoremata posterioris.

## *De Geometria*

168 This should probably be *Automatopoieticum*.

## *Cap II*

Geometriae quinque dari species supra docuimus, quae cum iisdem seiunctim insistant principiis, iisdemque utantur tum Problematis, tum Theorematis quibus ipsum genus: propterea secuti Mathematicae principem, agemus tantum breviter de Geometria, eiusque discendae docendaeque viam generali quadam ratione commonstrabimus.

## *Quae conferant ad discendam docendamque Geometriam*

### *Dist I*

Quinque sunt ad discendam docendamque Geometriam necessaria: finitiones, principia quae positiones dicuntur, petitiones, //8 Theoremata & Problemata: quorum postrema duo (quae uno nomine propositiones adpellantur) e reliquis tribus originem trahunt: principia quanquam priora sunt in Mathematicis quorum principia sunt: quoniam tamen ut plurimum finitiones mutuas sumimus a Physicis, que ad petitionum positionumque formam conducunt: ideo finitiones primo, secundo positiones, tertio petitiones Geometricas explicabimus, dein ad propositionum finitionem, necessitatem, situm, locum, naturamque deveniemus.

### *Definitionibus Geometricis*

### *Dist II*

Punctum est principium continui, cuius pars nulla. Puncta lineae non sunt partes, sed termini. Linea est quantitatis longa, non lata, fitque ex lineis. Lineae superficiem non constituunt, //v sed terminant. Superficies est quantitas longa lataque, non alta, quae fit ex superficiebus.

Superficies corpus non construunt, sed sunt extrema corporis.

Corpus est quantitas longa, lata, atque alta, quae fit ex corporibus.

Recta linea est puncti ad punctum brevissima extensio.

Duae rectae lineae superficiem non concludunt, quare neque figuram faciunt.

Parallelae duae lineae sunt, quae cum in eodem sint plano, non concurrunt, etiam in infinitum ductae.

Planus angulus est duarum linearum in plano, non iacentium indirectum, alterius ad alteram inclinatio.

Angulorum treis sunt species, rectus, obtusus, & acutus.

Quod si recta in rectam cadens duos angulos aequales fecerit, uterque rectus erit, & cadens perpendicularis adpellabitur.

Anguli rectilinei sunt in quadruplici //9 differentia, mutui sive deinceps, alterni, verticales, & interne vel externe oppositi.

Figura duplex est, plana & solida.

Plana est superficies terminata: cuius duo sunt genera, simplex et multiplex.

Simplex, ut circulus, qui ab una linea describitur, unoque puncto, quod centrum est.

Multiplex, ex laterum multitudine nomen ducit.

Trilaterarum figurarum treis sunt species, Isopleurus, Isosceles, & Scalenum: vel enim aequalis sunt anguli, aequaque latera: vel aequi, qui ad basin, anguli, duoque aequa latera: vel inaequalia omnia.

Basis est latus cui figuram inniti volumus.

Rursum treis sunt trilaterarum species, rectangula, obliquangula, vel acutorum omnium angulorum.

Quadrilaterarum quinque sunt species. Quadratum, Parallelogrammum //v Rhombus, Romboides & Trapezium.

Recta linea in circulo congruere dicitur, cum eius extrema in circuli circumferentiam cadunt.

Circuli diameter est recta linea per centrum extensa, quae in circulo congruit.

Parallelogrammi rectanguli diameter est recta ab angulis oppositis, tamquam terminis, excitata.

Parallelogrammi rectanguli ea quae per medium diameter secat parallelogramma circum eandem diametrum dicuntur consistere.

Parallelogrammi rectanguli partes parallelogrammae circa diametrum non consistentes, adpellantur supplementa.

Consistentium vero pars altera, utroque adscito supplemento, gnomon nuncupatur.

Pars circunferentiae circuli, arcus est.

Sector circuli, figura est quae sub duabus a centro ductis lineis, & arcu interiacente continetur.//10

Angulus ad centrum, qui ab eiusmodi rectis concluditur.

Angulus autem ad circumferentiam, cum duae rectae arcum comprehendunt.

Sectio circuli, figura est sub recta & arcu comprehensa.

Angulus sectionis, qui sub recta & arcu continetur.

Altitudo figurae est a puncto verticis ad basin ducta perpendicularis.

Angulus solidus est, qui sub pluribus suobus planis angulis comprehenditur in una non existentibus superficie, ad unum tamen punctum constitutis.

Solida figura est corpus terminatum: cuius regulares quinque sunt species, Sphaera, Cubus, Pyramis, Conus & Cylindrus.

Sphaera est arcus semicirculi circum fixam & immotam suam diametrum, donec ad primum locum revertatur, circumductus.

Cubus est figura solida sub sex quadratis contenta lateribus.//v

Pyramis est figura solida planis comprehensa ab una superficie ad unum signum constituta.

Conus est, quando rectanguli trianguli manente uno eorum, quae circa rectum angulum, latere, circunductum triangulum, in idem rursum, unde sumpserat initium, revolvitur.

Cylindrus est, quando rectanguli parallelogrammi manente uno eorum, quae circum rectum angulum, latere, circumductum parallelogrammum in idem, unde sumpsit exordium, steterit.

***De principiis seu positionibus***

***Dist III***

Quae uni tertio, vel ipsi aequo aequalia sunt, vel in eadem cum ipso, aut ipsi aequo, ratione constituta, inter se sunt aequalia.

Si figura figurae adplicata, vel facie corporis faciei, altera alteri, ipsa sibi mutuo congruant, aequales quoque sunt superficies, & aequa corpora.//11

Si ab aequalibus aequalia deducantur, vel ad ipsa aequalia accedant, tota seu reliqua sunt aequalia: & si inaequalia, inaequalia.

Si ab inaequalibus aequalia deducantur, vel ad ipsa aequalia accedant, tota seu reliqua sunt inaequalia.

Si duae rectae coniungantur in puncto, in ea parte contactus erit, in qua recta quaedam utranque dispescens, internos & oppositos duobus rectis minores faciet.

Aequae mutuaeque diametri aequales faciunt regulares figuras.

### *De postulatis seu petitionibus*

### *Dist IIII*

Poscimus ut a puncto in punctum recta duci possit, eaque in infinitum protrahi: & ab ea minor deduci.

Et quocunque puncto & intervallo circulus describi.

### *De propositionibus*

### *Dist V* //v

Propositionum duas esse species diximus, Theorema & Problema, quarum prior contemplatur, posterior in opere est: neutra vero sine altera consistit, utraque ex finitionibus, principiis & petitionibus nascitur, atque adeo paulatim excrescens, facit ut admirabiles infirmiori naturae praestet operas, a quibus altius iam provecta, universam mundi fabricam perscrutatur.

### *De Theorematis & Problematis*
### *finitionibus et natura*

### *Dist VI*

Theorema dicitur propositio, aiens, docens, conditionalis, cuius finis est sola doctrina vel cognitio: Problema, propositio agens, absoluta, imperans cuius finis est opus ipsum. Prima Euclidis propositio problema est: super data recta linea terminata aequilaterum triangulum constituere: mandat enim ut triangulus Isopleurus describatur super recta, hocque agi prae //12 cipit adcurate: propositio quarta Theorema est: si duo trianguli duo latera duobus lateribus aequalia habeant, alterum alteri, & angulum angulo aequalem sub aequalibus rectis comprehensum, aequilateri sunt & aequianguli: docet enim & adfirmat duos triangulos aequilateros & aequiangulos esse, sub conditione quidem, si nimirum duo latera duobus lateribus aequalia habeant, alterum alteri, ac praeterea angulum angulo aequalem sub aequalibus rectis contentum: atque huius propositionis finis est una cognitio: nanque nil quicquam iubet fieri, sed docet, sub conditione adserit, & cognoscit.

### *De propositionum necessitate & situ*

### *Dist VII*

Nobilis fuit inter Mathematicos controversia, utra propositionum species situ necessitateque sit illustrior: Problemane Theorema superet, vel idem Theorematі post//v ponatur, ac certe bona illorum pars rationem secuti, quod cognitio actionem debeat praecedere, Theorema prius esse Problemate confessi sunt: pars altera auctoritate nixi contrarium adseruerunt: in hoc omnes conveniunt, quod Mathematicae necessitas ex propositionum necessitate petitur: caeterum ego facilis iuero[169] priorem in sententiam, nulla contemplatione, quod Euclides a Problemate principium duxerit, cum idem ipse potuerit a Theorematis exordiri, prorsus immutato docendi ordinem quod quisque paulum in Geometricis provectus deprehendet.

### *De propositionum loco*

### *Dist VIII*

Geometricarum propositionum infinitum esse agmen non est qui dubitet, cum scientiae infinitum sit obiectum, puta continua quantitas, ipso teste Philosopho: verum divinus // 13 Euclides acute Mathematicis consultus, hanc, ut ita loquar, ad Daedalum viam aperuit, partumque Mathematices informem, lambentis Ursae instar, tantisper admirabilis explicuit Theon, donec efformavit: quo fit, ut qui Euclidem magistro

169 This should probably be *invero*.

Theone fuerit adsecutus, amplissimo donetur Mathematicorum munere, hoc est nomine immortali.

## *De ternario Geometrico*

### *Dist IX*

Admirabile est in Mathematicis arcanum, ternarii vim atque potestatem nosse, quae hactenus vestigata, nec inventa, si quis prima pietate doctrinaque homo deprehenderit, neque tamen summum Geometricorum opus iuvantibus excitaverit Mechanicis, plus profecerit ille, quam omnes una Philosophi: enimvero non iniuria Plato Eudoxum, Archytam, & Menechnum insignes Philosophos arguebat, quod Geometricum ternarium ad sen //v silia revocarent, nimirum hac contemplatione Mathematicen deperdentes: ergo suscitemus ipsi nos de reliquarum disciplinarum somnis, laboremusque semel in eo, quod, radiis in arena adparentibus, simus homines, quod, investigatis secundum oraculum Cubicis problematis, Deorum similes existamus: has denique propositiones Mathematici demonstremus.

## *Propositiones hactenus in Geometricis desideratae*

### *Dist X*

- Datum angulum rectilineum in treis aequas partes dividere.

- Isosceles triangulum constituere, quod habeat angulos ad basim triplos reliqui.
- Datis duabus rectis lineis duas intermedias proportionales deprehendere.
- Datae circuli circumferentiae aequalem rectam lineam invenire.//14

//14 ***De Arithmetica***

***Cap III Dist I***

Duplicem Arithmeticae divisionem supra tradidimus, priorem in tria membra, posteriorem in duo secuimus: cum vero taediosum sit utramque partitionem singulatim percensere, nec instituti nostri quosque expedire canones, sed illorum expediundorum rationem commonstrare, missa priore, & posteriore cognoscente ut quae maxime Geometriae adfines, cuius discendae docendaeque viam dedimus, descendemus ad posterioris speciem alteram, quam practicam nominant, agentem tu dixeris: huiusque principia, petitiones, axiomata, & propositiones recitabimus, diversas quidem a parte altera cognoscente, atque adeo a Geometria, si modo lineatum in numerum converteris.

***De rudiore Arithmetica agente***

***Dist II*** //v

Agentis arithmeticae cardinales partes quatuor sunt, Adductio, Deductio, Productio, & Diductio: quarum functio triplex est, absolutorum, particularum, & rationum: progressio sequitur adductionem & deductionem: proportionum regula productionem & diductionem: Hypothesium uterque canon membris omnibus nititur: quae ad Laterum educendorum rationem conferunt: haec nos omnia perstringemus breviter, docebimusque immensum rudioris Arithmeticae pelagus, hoc problematum numero, tanquam tutissimo litore, coerceri.

### *Rudioris Arithmeticae finitiones*

### *Dist III*

Monas est, qua quaeque res una dicitur.

Numerus, monadum multitudo. Numerus alium numerare dicitur, qui secundum aliquem multiplicatus illum genuit. //15

Primus numerus, quem sola monas numerat.

Compositus, quam numerat alius a seipso numerus.

Primi numeri dicuntur, qui a nullo communi numero numerantur.

Compositi, qui ab aliquo.

Particula est aliquis minus monade: puta unus quadrans, duo trientes.

Particulae, aliquid est maius monade, absoluto minus: puta treis semisses, undecim trientes, cum supereunt ternarium, deficiant a quaternario.

Pars est numerus numeri minor maioris, quando minor maiorem numerat.

Partes, quando non numerat.

Perfectus numerus est, qui sui ipsius partibus est aequalis.

Ratio, pars est vel partes, particula vel particulae.

Proportio, partium ad se invicem comparatio, partium ratio, sive rationum identitas.

Numeri proportionales dicuntur, quan//v do primus eadem est pars aut partes secundi, quae pars aut partes tertius quarti.

Productio nihil est adductionis simile, nec diductio deductionis.

Latera distinguntur a radicibus, quemadmodum lineae a superficiebus.

Quod prodit ex diductionis opere, dicitur Parabola.

Quod particulam particulasve numerat, superque scribitur, appelatur numerator.

Quod particulam particulasve nominat, infraque scribitur, vocatur nominator, sive nomen, seu notator.

Rationum species hae sunt praecipuae, aequa, conversa, permutata, composita, divisa, ordinata, & perturbata.

Quadratus numerus est, qui sub duobus aequalibus numeris continetur.

Cubus autem, qui sub tribus: qui numeri aequales latera dicuntur.

### *Petitiones*

### *Dist IIII* //16

Cuilibet numero aequales posse sumi, vel multiplices ipsoque maiores, vel minores.

Seriem numerorum in infinitum posse produci.

Nullum numerum in infinitum posse minui.

### *Propositiones*

### *Dist V*

Diversa nomina ad unum idemque revocantur, ducto numeratore particulae prioris in nomen posterioris, et vicissim numeratore posterioris in prioris notatorem, denique nominatoribus in se invicem.

Adductio, deductio & diductio particularum, reductis nominibus, sunt eadem: rationum deductio prorsus est diductio particularum.

Productio particularum fit ductis numeratoribus & nominibus invicem: qualis est adductio rationum.

Proportionum regula ducto secundo //v termino in tertium, factoque per primum diducto, quartum suppeditat: nec absimili ratione reliquos, separatim ductis reliquis, elicit: primo & tertio unius semper generis existentibus.

Simplicis Hypothesis canon sumpto aliquo numero, atque ex eo perfecta quaestionis ratiocinatione, proportionum regulam sequitur.

### *Theorema ad usum Canonis duplicis Hypothesis*

Si pro ignoto quaestionis alicuius numero duo quilibet adsumantur numeri, & ex utroque seiunctim quaestionis formula pertractetur, ac si quid vel supersit demum, vel desit, cum nota redundantiae, vel defectus adscribatur, erit sicut differentia errorum operis ad utrumlibet ipsorum errorum, sic differentia Hypothesium ad errorem eius Hypothesis, cuius erratum secundum proportionale sumptum est: quod Hypothesis erratum, Hypothesis vel additum, //17 siquidem hypothesis fuerit minor atque oportuit, vel detractor, si maior, quaesitum suppeditat numerum.

Unde constat utrumque Hypothesis genus ex falso verum investigare.

### *Theorema ad quartum secundi Euclidis, numeris adplicatum, & generale factum, ad usum educendi Quadrati lateris*

Si numerus in quotcunque partes dividatur: Quadratum totius aequale erit Quadratis partium, & duplo facti ex unaquaque partium in reliquas.

### *Theorema superiori proportionale, ad usum eruendi lateris Cubici*

Si numerus in quotcunque partes dividatur: Cubus totius aequalis erit Cubis partium, & facto ex unaquaque in triplum Quadratorum reliquarum: quod, quam verum est, tam intellectu difficile, si non adverteris.

Productionis & diductionis rationum una est via, sed altera alteri contra //v ria: si quidem per binarium fiat productio, multiplicentur Quadrate termini, si per ternarium Cubice: et ita deinceps. Si per binarium fiat diductio, terminorum eruantur Quadrata Latera: si per ternarium, Cubica: si per quaternarium, Quadratiquadrata: sicque in infinitum.

## *De subtiliore Arithmetica*

### *Cap. IIII. Dist I*

Singulare est in Quadratis Cubicisque nominibus, quibus suis terminis rudior Arithmetica concluditur, subtiliorem hanc, tanquam in simplicibus numeris, operari: puta non aliter Quadrata quatuor dignoscere, quam per quaternarium: non secus treis cubos, quam per ternarium: illudque adeo, quod rudioris extrema constituit, principia facere subtilioris: quae licet infinita, praecipua tamen sunt Latus, Quadratu~, Cubus: finis scientiae, quantitatis ignoratae cognitio: media ad illum finem, aequatio, vel aequalitas. //18

### *Finitiones. Dist. II.*

Numerus est solis monadibus conflata multitudo: cuius species sunt quatuor, absolutus, particular, rationalis, et rationis expers.

Nomen est terminus in continua proportione Geometrica, secundum octavae noni Euclidis ordinem, constitutus.

Valor autem nominis, terminorum series.

Particula vel particulae sunt cum uterque, vel notator solus, nominibus insigniuntur,

Aequatio dicitur, cum aliquae quantitates diversi generis inter se aequales proferuntur.

Simplex, cum duae solae.

Composita, cum plures uni, pluribusve.

Aequatitia, cum ex operis necessitate fit aequatio.

Fictitia, cum ad arbitrium Arithmetici consistunt hypotheses.

Aequalitas duplicata dicitur, duarum aequationum per quantitates inter//v vallum aerum multiplicantes, ad simplicem reductio.

Emendata aequatio est, quando aequationis termini, qui rationis expertes erant, ad rationales, substitutis aliis Hypothesibus, revocantur.

Interaequatio dicitur, cum in aequatione emendanda, duos inter numeros nomen aliquod est inquirendum.

Adaequatio dicitur, quando in inveniendis ad problematis explicationem nominibus, certi cuiusdam numeri Lateri proximo, tanquam vero, nostras adplicamus hypotheses.

## *Petitiones*

## *Dist III*

Poscimus, ut si quantitas de quantitate diversi nominibus subduci debeat, huic adscripta cum nota defectus, subducta intelligatur: penuriae vero nota esto - .

Quantitas, quae signum penuriae non habuerit, haberi intelligi signum coptae: cuius nota esto + .//19

Signum + deductum ex signo - : signumque - deductum de signo + suam (sic?) relinquere cum nota redundantiae.

Signa eadem inter se multiplicata, vel deducta, facere signum copiae.

Signa vero diversa, producere signum penuriae.

Et quantitates diversi nominis addi vel deduci per signa + et - .

### *Propositiones*

### *Dist IIII*

Diversa nomina invicem multiplicantur sumpto, secundum valorum summam, nomine: deducuntur autem, sumpto, secundum differentiam, nomine.

Latera diversi nominis multiplicantur, ipsorum numeris inter se, cum prius ad idem nomen revocata fuerint, multiplicatis, sumptoque producti Latere: diducuntur autem, ipsorum diductis inter se numeris, sumptoque latere parabolae.//v

Latera diversi nominis adduntur, prius ad idem nomen revocata, partito maiori numero in minorem, parabolae Lateri communis nominis addita monade, summaque per minus Latus multiplicata: deducitur autem, a parabolae Latere monade detracta, residuoque per minus Latus multiplicato.

In aequatione simplice consequenda, partiemur numerum eius quantitatis, quae minus nomen tenet, in numerum quantitatis maioris nominis, minusque nomen ex maiori deducemus, secundum quod nomen reliqua parabola numerorum explicabitur habebiturque quaesita quantitas.

In expediunda fictitia aequatione, quaesitum nomen efformabitur ab Latere, aliquo numero Numerorum, + vel - aliquot monadibus: aut ab aliquot monadibus, + vel - aliquo Numerorum numero: ea cautione, ut tandem una species diversi generis speciei adplicetur, simplexque consistat aequatio.//20

In tractanda duplicata aequalitate, duarum aequationum intervallo conspecto, duos perquiremus numeros, qui illud intervallum sua multiplicatione producant: horum vel intervalli semissis Quadratum aequale est quantitatibus minoris aequationis vel summae semissis Quadratum quantitatibus maioris: oportet autem sic exhiberi numeros intervallum illud multiplicantes, ut intra simplicis aequationis limites consistant.

Compositae aequationis, quando numerus aequalis est Quadratis et Lateribus, primus canon est eiusmodi: ut numerum in Quadratorum monades multiplicemus, facto semissis numeri laterum Quadratum adiiciamus, de<ni>que summae huius Latere semissem numeri Laterum deducamus, reliquum in Quadratorum monades diducamus: parabola quaesitum erit Latus.

Compositae aequationis canon secundus, quando numerus & Latera aequan//v tur Quadratis sic explicabitur: ut numero in Quadratorum monades multiplicato addamus Quadratum semissis numeri Laterum summaeque huius Latus Quadratum ad semissem numeri Laterum adducamus, summam denique hanc partiamur in Quadratorum monades: parabola vestigatum erit latus.

Compositae aequationis canon tertius is demum est cum Latera aequalia sunt Quadratis et numero: tum vero factum a numero Quadratorum in monades deducemus ex Quadrato semissis numeris Laterum, residui Latus Quadratum addemus vel deducemus ex semisse numeri Laterum, summa vel residuum divisa per Quadratorum monades ostendunt quaesitum Latus.

Quas superiores treis propositiones, et his proportionales, quinta secundi Euclidis, et tricesima primi Diophanti suis confirmant demonstrationibus.//21

***De ternario Arithmetico,***

***propositiones hactenus desideratae***

***Dist V***

Aequationes, quae sunt inter Latera, Cubus et numerum; Quadrata, Cubos et numerum; Quadrata, Latera, Cubos et numerum, notas facere.

Haec nos (Viri clarissimi) lusimus: et quanquam multas propositiones hic in Mathematicis fecimus desiderari, bona tamen illarum pars, imo, si paralogismum non adverterim, universa penes nobis est: neque vero per rationem licet ingenii vires maximas his in rudimentis ostendere:

committantur mea haec inventa singulis Vaticani Diophanti libri tredecim: iamque enim Vietaeum collegam vestrum, Cuiacium et Hollerium, senatores amplissimos, nobilesque Mathematicos pendentes animi video expectatione rerum Diophanticarum. Nempe (Viri quos Musae omnes //v amant) graecorum in Mathematicis eruditissimum, totum refertum mendis, a Gosselino repurgari posse credidistis: atqui nefas sit vestram de me non inglorio opinionem fallere: itaque (quod omen faustum sit) propediem animi vires explicabo.

FINIS[170]

170 The text of the booklet continues with a new short work:

Carissimo viro Carolo Cantoclaro supplicum libellorum in regia Magistro Gul. Gos. S.D.
Explicatus Cicero libro quinto ad Atticum epist. ultima
M. Scaptius et Salaminii
Si meo Bosius singulari vir doctrina fuit, ingenio admirabili...

This text is constituted by calculations useful for the interpretation of the work by Cicero. It does not contain algebra. What is relevant for us, as we lack information about Gosselin, are the contemporaries cited.

**Translation of the text**

## Guillaume Gosselin *de Caen en Issé*.

# How to learn and how to teach mathematics. Lecture.

To the most illustrious men
Jean Chandon and Charles Bocher,
*maîtres de requêtes* at the Court.

Almost all recent authors when they wanted to explain mathematics found themselves involved in the complexities of its parts, so that they did not classify the canons without mistakes, but introduced rules and forced this most noble science into the most narrow boundaries. They did it in such a way that the student could understand nothing. Actually, to say how things stand, having followed a complete course of mathematical studies, I have been exhausted by their books, and if their work had been destroyed just off the press, it would resurrect under better auspices.

*The skilled Apollo among many thousands of people.*[171]

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171 See the entire Latin text in the footnote. The whole passage means "The good and learned man, such that the skilled Apollo only found one among many thousand people, is a severe judge of himself, and explores the slightest details of his soul." Here Gosselin seems to indicate that we should face the task of going beyond the obstacles of ancient mathematics, keeping in mind the ideal of the self-conscious *vir bonus et sapiens*. This quotation is from the ***Appendix Vergiliana***. It would be interesting to know whether Gosselin attributed it to Vergil. We can see that this quotation, as well as the next, is not precise. It is a good example of a quotation by memory, i.e. not *verbatim.*

That is (to speak as a Platonist) we should not avoid mathematics, in the way of Apollo's followers, but on the contrary, as far as possible we should use absolutely all our mental faculties in order to understand the universe; but a difficult access almost blocked the way, and, as it were, the dark entrance terrified the bleary-eyed student. Taking these things to hearth, in numbers I have provided something easier than the rest but just as learned: so that both the beginner and the expert bring back what each of them wanted, and the author need take second place to no one. Only one thing is still wanted from a decent mathematician, according to what experience taught me, that he deals about some easy foundations, as soon as possible, according to what experience has taught him.

Therefore, as soon as I had a moment away from business, I found a scheme of how to teach and how to learn mathematics. When that work was examined more carefully, it appeared at first glance that it deserved publication, so that mathematical sciences could reveal themselves thank to this guide. Finally I was so much taken by sweet love towards the students, that I could not repress it. Furthermore, according to your wishes, it happened that the work is going to be published and communicated orally to you. Please, take this as a first and eternal proof of my homage toward you, and love us and its author, though for him it is true what Horace writes:

*It is not worth it. So I'll play a whetstone's part,* *which makes steel sharp but of itself cannot cut.*[172]

Best regards.

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172 The translation is from H. Ruston Fairelough, Horace ***Satyres, Epistles and Ars Poetica***, Loeb, Cambridge M.A. and London, 1978 (1926). This passage from Horace concerns the poet. The poet has to be mad in order to be creative, according to one of the platonic traditions about poetry. With reference to this idea, Horace had ironically defended himself from the accusation of not being a good poet. Certainly - he wrote in the previous verses -- if I were mad I would be the best of poets. But it is not worth it. So, too bad, I shall be like the whetstone, I shall just teach how to be a poet, without being one myself. The following verse is: "*munus et officium, nil scribens ipse, docebo*." Gosselin expresses modesty instead of Horace's irony. He seems to say "I just teach how to teach and learn mathematics, even though I might not be a great mathematician."

## How to learn and how to teach mathematics

***lecture***

That doctrine, discipline and science are one and the same thing.

### Chap.1 Sec.1

If there were no teaching there would be no learning;[173] with no learning, culture would be silent forever. This is why, even when teaching develops, if there is no learning, science is taken away. For, when an intellect well-formed in particular things acknowledges in the reciprocal harmony of sensible things a universal notion, it instructs itself, and learns from itself: from which genus of beings we remove the *doctrina* of the sense, which we call by the foreign name "apprehension," so that the terms of doctrine and discipline will be included in one notion, and therefore will have an essential connection with science.

### Of what kind of doctrine is mathematics

### Sec. II

173 Already from the title, Gosselin shows that he is thinking within the framework of the ***Posterior Analytics***.

The human being has three functions: sense, intellect and phantasy, which has something in common with the first two. Sense deals with the individual, as present; phantasy with the individual, as absent; the intellect, sometimes with present, sometimes with absent individual things, insofar as they are composed, like a sort of universal feature. For apprehension belongs to sense, imagination to phantasy, but only intellect is subordinated to discipline. Leaving aside the other two, we shall see how many kinds of disciplines, hence whatever thing is understood by the intellect, is admitted either by demonstration or by faith or is confirmed by authority. From this we deduce that any doctrine is either philosophy, divine theology, or history. Omitting two of them, we shall examine that science that we call philosophy. It divides into five parts, for either it teaches the subtleties of discussion, or is concerned with obscure natural questions, or defines life and customs, or deals with being qua being, or finally shows the features of quantities, and this is what is called mathematics, and it is here we begin our discussion.

## Which degree of science is proper to mathematics

## Sec. III

It is clear that all teaching and all learning are constituted by a previous knowledge of principles,[174] and that principles of science are known only as presuppositions; a demonstration therefore originates from its presuppositions. Its effect is all the more pure in that it proceeds from definition, because definitions can be demonstrated by going back indefinitely and each definition has a definition, whereas there is no principle of a principle. Furthermore, of two sciences, that science is more demonstrated which rests on principles rather than on definitions, given that principles are obviously more evident; therefore it will be taken for confirmed that such a science is more demonstrated. Given all this, it is fitting that the mathematical part of philosophy (consisting of five parts), which rests and operates better on more known principles, rightfully claims over the others the name of doctrine, science and discipline; and this in spite of the fact that the evil contingent of nominalist philosophers[175] all agree in stating that metaphysics is the most certain of all sciences, given that it is founded neither on the thing, nor on the principle of physical matter, for mathematics, they say, "separates from" in thinking, but not in reality. But these people are clearly mistaken insofar as, the "intensio" of

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174 This is, of course, a paraphrase of the beginning of the ***Posterior Analytics***. There were a few editions available in Paris at the time.

175 Here Gosselin refers to the nominalist tradition, which from Roscellinus on recommended that the *universalia* be considered only in connection with the mind. Gosselin, however, is likely to have thought of Alessandro Piccolomini who, in 1547, proposed to interpret Averroes' commentary on the ***Metaphysics*** in a new way. Mathematics is certain not because it uses the scientific reasoning, which it doesn't, but because its object is in the mind. See, on sixteenth century discussions on this topic, G. Crapulli 1969 and P. Dear 1988.

science, so to speak, is taken as from the object, not from the subject.[176] So, even if metaphysics underpins the thing in neither case, in the second case, mathematics underpins it, as in the triangle with respect to the wood. For it is not by assuming this or that triangle, but rather by taking a general case for this or that subject that we conclude according to the surest principles that all triangles have three angles equal to two right angles.

## What is the object of mathematics

### Sec. IV

There are those who are convinced that the figure as a species of quality is an object of mathematics.[177] In this case they are totally wrong, because, while interpreting Aristotle, they strayed from him. For according to Aristotle figure means two things: it can be considered either with respect to the threefold aspect of the continuous quantity, or by means of the distinction of terms; in fact, in the second case, figure is a quality, in the first case it is a quantity. Insofar as they indiscriminately made figure qua figure the object of mathematics, they made the object of mathematics false and empty. Actually, in our opinion, one cannot deduce directly from the fact that figure is a quality the conclusion that it should be defined as an

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176 This is the traditional interpretation of several passages of Aristotle's ***Metaphysics***, in particular 1077b and 1078a.

177 In fact, this is the interpretation of ***Categories*** 10a11 which was common in the XVth and XVIth centuries, in particular with reference to the quadrature of the circle. See N. von Cues ***Mathematische Schriften***, ed. by J. E. Hofmann, Hamburg 1952.

object, but only that it belongs to the object, so that figure can belong to the object of mathematics. In order to make this clear, let us take number as an example. And there is no better reason for the figure to be an object [of mathematics] than for number, for, in the same way as figure is proper to geometry, number is proper to arithmetic and each of these parts is an equal part of mathematics. Therefore, having refuted the above position of philosophers and mathematicians, everyone will agree that quantity is the object of mathematics, and that is not reducible to the art of figures.[178]

## What are the species of mathematics

### Sec. V

It is certain that the same object gives rise to distinct species, and that precisely the proper object of mathematics is that same quantity that creates the second supreme Being.[179] In fact, two differences divide this being, the conjoint and the disjoint and again they split into two subordinate genera: one including length, width and height; the other, in which number is included; the first is called geometry, the second arithmetic. And it is not

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178 In this way, Gosselin seems to restrict the proper domain of mathematics to geometry on one hand and to arithmetic and an arithmetical treatment of quadrature on the other hand.

179 This is the first explicit reference to Proclus, whose commentary on Euclid had been made famous in Paris by Ramus' ***Scholae mathematicae***. Proclus, at the beginning of his treatise, gives this neo-platonic and theological foundation of mathematics. From this point on, the partition of topics is largely inspired by Proclus.

possible to invent more genera for mathematics, since everything which is quantum is either quantum linearly, or quantum numerically:[180] the linearly simple <quantum> <is for instance> the circle, among plain figures, or the sphere among solid figures; the quantum linearly composed <is for instance> the square among plane figures,and among spherical figures and bodies, the cube.  An example of the quantum numerically is the ternary among the absolutes divided into parts: and in this way we have created two genera of mathematics and two different species according to the object, geometry and arithmetic.  Geometry deals with the continuum as such.  In fact, there is nothing concerning motion, place or time under the name of continuum, given that motion is transmitted by ten universal beings and it must belong to that to which it is applied: the place would not be different from the "essence of surface" if it was a part of its boundaries. Finally time would not exist without the instant, because only indirectly thanks to the instant can it be traced back to the continuum; just as the point, the monad and the mutation are reduced <to the one>.  Arithmetic considers the number as such, hence we do not consider it fitting to put the argument so that we remove the infinite and separate the numbered from the number.  There are already many kinds of each sort of quantities, insofar as the continuous and the discrete participate in many relations: the continuum takes many forms: terrestrially, it is the foundation of geodesy; celestially, of astronomy, in the form of either air or water it is the basis of

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180 Everything has a size, or can be numbered.

pneumatics, in weight, the basis of the automatics, and visually, the basis for optics, or finally in mixed form, for Mechanics. The discontinuous also presents itself in many forms, because of the three proportions of numbers: the ratio is understood in terms of excess of equal terms <arithmetic progression>, which makes rougher Arithmetic, or <in terms of> multiples <geometric progression>, which makes subtler Arithmetic or a proportion, mixing equal and multiple terms, as in harmony. Finally both species of quantity divide into two, acting and knowing. The latter gives laws and defines rules, the former, resting on knowledge, completes the action that it pursues; problems belong to the first, theorems to the second.[181]

## Cap.II Geometry

We have explained above that there are five species of geometry, which, while they are founded separately on the same principles, make use of the same things, both in problems and in theorems, as the genus itself; thus, having followed the prince of mathematics, we shall briefly deal with geometry and we shall find a path for teaching and learning it in a general way.

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181 On this distinction between problems and theorems, its Greek and sixteenth century tradition, see the Appendix and its references.

## What is conducive to learning and teaching geometry

### Sec. I

Five things are necessary to learn and to teach Geometry: definitions, those principles which are called hypotheses, postulates, theorems and problems; the last two (which are called by the simple name propositions) take their origin from the first three: even though principles have priority in mathematics, of which they are the foundation [principia] nonetheless, since we have borrowed our definitions from physicists, and these definitions lead to the two forms of assumption and presupposition, so we shall present first the definitions, then the presuppositions, finally geometrical assumptions, and then we will come to the definition of propositions, and to their necessity, position, place, and nature.

## Geometrical definitions

### Sec. II

The point is the principle of the continuum, and has no parts. The points are not parts, but boundaries of a line. The line is, in terms of quantity, long, not large, and is made out of lines. Lines do not constitute but delimit surfaces. A surface is a quantity long and large, not high, and is made of surfaces.

Surfaces do not constitute a <solid> body, but are its boundaries. A body is a quantity which is long, large, high, and is made of bodies.

A straight line is the shortest path of a point to a point.

Two straight lines do not close a surface, and do not constitute a figure.

Two parallel lines are those which are in the same plane and do not touch, even when drawn to infinity.

A plane angle is the reciprocal inclination of two lines in a plane which do not coincide.

There are three species of angles: straight, obtuse and acute.

If a straight line intersecting with a straight line makes two equal angles, they will both be straight angles and the line will be called perpendicular.

Rectilinear angles are of four sorts, i.e. reciprocal, alternate, vertical and internally or externally opposite.

Figures are of two sorts, plane and solid.

A surface with boundaries is plane, and there are two sorts, simple and multiple.

Simple, like the circle, which is described by a single line, and by a single point, which is the center.

Multiple, which takes its name from the number of sides.

There are three species of trilateral figures: Isopleurus, Isosceles, Scalenum. For either the angles are equal, and the sides are equal, or both the angles at the basis and two sides are equal, or they are all unequal.

The basis is the side on which we want the figure to rest.

In turn, there are three species of trilateral figures: rectangular, obliquangular or with all acute angles.

There are five species of quadrilateral figures: the square, the parallelogram,the rhombus, the romboid and the trapezoid.

The straight line is said to be congruent in a circle, if the ends of the line are on the circumference.

The diameter of a circle is the straight line extended through the center, which is congruent in that circle.

The diameter of a rectangular parallelogram is a straight line drawn from the opposite angles, taken as limits.

In a rectangular parallelogram, what is divided in two by the diameter when it cuts through the middle is said to form a parallelogram around the same diameter.

The parts of the rectangular parallelogram which are not around the diameter, are called supplements.

The second of the existing parts, after one supplement is added, is called gnomon.

A part of the circumference of the circle is an arc.

A sector of the circle is a figure contained under two lines drawn from the center, and the arc between the two.

An angle at the center is the one included by straight lines of this kind.

An angle at the circumference is made when two lines include an arc.

A section of the circle is the figure contained between a line and an arc.

An angle of the section is contained between a straight line and an arc.

The height of a figure is the perpendicular drawn from the point of the vertex to the basis.

An angle is solid when it is included in its multiple plane angles, which are not on the same surface, but converge in a point.

A solid figure is a bounded body. There are five species of solid figures: the sphere, the cube, the pyramid, the cone, and the cylinder.

The sphere is the arc of a semicircle rotated around a fixed and unmoved diameter, until it returns to its original position.

The cube is a solid figure contained by six squared sides.

The pyramid is a solid figure delimited by planes and determined by a surface and a fixed point.

The cone is produced by a rectangular triangle when one of the sides of the straight angle is fixed, and the triangle is rotated until it returns to its original position.

The cylinder is produced by a rectangular parallelogram when one of the sides of a straight angle is fixed, and the parallelogram is rotated until it returns to its original position.

## On principles or presuppositions

### Sec. III

All those things which are both equal to a third, or to itself or to themselves, after having established a ratio, are equal in a ratio among themselves.

If, after applying one figure to another, or a face of a body to another face, one by one, they are mutually congruent, the surfaces are also equal, and the bodies are equal.

If equalities are deduced from other equalities, or equalities are similar to themselves, all the remaining things are equal, and, if unequal, unequal in the same way.[182]

If equalities are deduced from inequalities or the equalities are similar to themselves, all the remaining are also unequal in the same way. If two straight lines converge in a point, the contact will take place in a part in which a straight line will cut the other in half, making internal opposite angles smaller than two straight angles.

Equal diameters make regular figures equal.

## Postulates or assumptions

## Sec. IV

We postulate that a straight line can be drawn from a point to another, and that it can be drawn to infinity, and that a smaller line can be subtracted

182 These first three assumptions [*positiones*] appear also in ***De Arte magna***, where they are called *Axiomata*. (pp. 54v-55)

from it. We also postulate that a circle is described by any given center and radius.

## Propositions

### Sec. V

We said that there are two sorts of propositions, theorems and problems, the first of which is theoretical, and the second is practical: in fact, neither of them exists without the other, since both derive from definitions, principles and assumptions. Gradually increasing, they make it possible to provide the weaker nature with remarkable works; having then been raised above these works, they investigate the whole construction of the world.[183]

## Theorems, problems, definitions and nature

### Sec. VI

A theorem is said to be a proposition, be it affirmative, instructive or conditional, the goal of which is doctrine or knowledge alone; a problem is

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183 In this and in the following section we see Gosselin developing a theory of problems, which are of great importance in the XVIth century French rethinking of mathematics and in view of a legitimation of algebra in the construction of a mathematical proposition. Two novelties are the results of this rethinking. First, the transformation of the notion of problem, so that eventually it combines the abacus' *question* with the classical mathematical problem. Second, the identification of problem with the general equation. See Appendix for a detailed account of this process.

said to be a proposition acting, absolute, or prescriptive, whose goal is the work itself. The first proposition of Euclid is a problem: to construct an equilateral triangle on a given bounded straight line. For, it proposes to construct an isopleurus on a straight line, and prescribes accurately how to do this. The fourth proposition is a theorem: if two triangles have two sides equal to each other, and the angle included between the same straight lines is equal, they are equilateral and equiangular. For, it teaches and states that two triangles are equilateral and equiangular if assuredly they have two sides equal to each other and especially if the angle included between two equal sides is equal. And the purpose of this proposition is only knowledge, for it does not prescribe anything whatsoever to be done, but teaches, asserts under condition, and knows.

## The necessity and status of propositions

## Sec. VII

There has been a noble controversy among mathematicians, about which <kind of> proposition is more important by necessity and status, whether the problem is more important than the theorem or vice versa.[184] Certainly, most of these mathematicians, having followed the opinion according to which knowledge must precede the action, maintained that the theorem

184 The dispute was at least as old as Greek geometry. See Appendix.

should precede the problem; another group, moved by authority, asserted the contrary. But they all agreed about this, that necessity in mathematics comes from the necessity of propositions. I therefore shall follow with no doubts the path of the first opinion, that Euclid began with the problem, even though he could have started from theorems, without having changed the order of teaching, which anybody minimally expert in geometry learns.

## The place of propositions

### Sec. VIII

Nobody could doubt that there is an infinite number of geometrical propositions, given that the object of science is infinite, for instance quantity is continuous, as the Philosopher himself states. In fact the divine Euclid, after closely consulting the mathematicians, showed the way to the labyrinth, so to speak. Later Theon admirably made explicit the formless product of the birth of Mathematics, giving it shape just as the bear gives shape to its newborn.

Thus, it happens that whoever, taking Theon as his guide, follows Euclid receives the huge patrimony of mathematicians, that is the name of immortal.

## The geometrical ternary

Sec. IX

There is a marvelous mystery in mathematics, i.e. the knowledge of the form and the power of three which has so far been investigated, not found. If someone had approached with the primordial piety and doctrine, and yet had not made use of the greatest works of Geometers, but only the help of Mechanical artisans, he would have accomplished more than all the Philosophers together. However Plato was right in criticizing the great philosophers Eudoxus, Archytas, and Menechmus, insofar as they reduced the geometrical ternary to the sensible, losing in this contemplation much of the mathematical. Therefore let us wake ourselves from the sleep of old ideas, and let us work at this once and for all, so that when using the sticks in the dust[185] we be men, and, after having treated the problem of the cubes according to the oracle,[186] we be similar to gods. Finally we shall prove these propositions as Mathematicians.

Propositions so far needed in geometry

Sec. X

185 I.e., when at work as mathematicians. Gosselin seems to use *arena* for *pulvis*, the green glass dust used to draw, with the *radius*, geometrical figures.

186 The Delian oracle, to duplicate the cubic altar to Apollo, in order to stop the plague. In fact, Gosselin will also refer, more explicitly in the next section, to the other two classical problems: the quadrature of the circle and the trisection of the angle.

To divide a given angle into three equal parts.

To construct an isosceles triangle such that its angles at the basis are three times the angles at the vertex.

Given two straight lines, to determine two mid-proportionals.

Given the circumference of a circle to find a straight line of the same length.

## Arithmetic

### Chap. III Sec. I

We have already divided arithmetic into two parts. The first is in turn divided into three parts, the second into two; given that it has seemed boring to deal with it separately, it is not the intention of our teaching to deal with the various canons, but to show the order in which to expound them. Leaving aside the first <arithmetic> and the second speculative <arithmetic>, insofar as it is most similar to geometry, and for which we have already given a way of learning and teaching, we shall treat the second kind, which is called practical, and you would call "acting." And we shall recite its principles, assumptions, axioms and the propositions which are different from the speculative part and from geometry, if only you change the line to a number.[187]

187 The identification, asserted twice in this section, between speculative arithmetic and geometry is what allows Gosselin to avoid a detailed treatment of speculative arithmetic and

The rougher acting arithmetic

Sec. II

There are four cardinal parts of acting[188] arithmetic: addition, subtraction, multiplication, and division. They have three functions: on the absolute <numbers>, on the particles <parts of one>, and on the fractions. Progression comes after addition and subtraction; the rule of proportions follows the product and the division; both canons of hypotheses are based on all members; all things that refer to the extraction of roots. All this we shall deal with, and we shall teach the immense sea of the rougher arithmetic, trying to limit ourselves to a certain number of problems as a safe harbor.

Definitions of rougher arithmetic

Sec. III

The monad is that by which anything is said to be one. Number is a multitude of monads. A number is said to number another when it

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to proceed directly to give definitions pertaining practical [*agens*] arithmetic, in particular, algebra.

188 This term derives from Ramus, who eliminates the distinction between art and science of the same object, and the distinction between theoretical and practical sciences. For, the same *mathesis* is *utilis ad contemplandum* or *ad agendum*, i.e. what varies is its use.

generates a number when multiplied by the other.

A prime number is a number numbered by only one monad.

A composite number is a number which can be numbered by a number different from itself.

Prime numbers are said to be those which cannot be numbered by any common number.

Composite numbers are those which can.

A particle is something less than a monad. For instance, one half, two thirds.

Of a particle, something is greater than a monad, but in the absolute it is smaller. For instance, three halves, eleven thirds, although they are more than three, they are less than four.

A part is a smaller number of a greater number, when the smaller numbers the greater.

Parts, when it does not number it.

A perfect number is a number which is equal to its own parts.

A ratio <fraction> is a part or parts, a particle or particles.

A proportion is a comparison of parts to each other in turn, a ratio of parts or an identity of ratios.

Numbers are said to be proportional when the first is the same part, or parts, of the second as the third is of the fourth.

Multiplication is by no means similar to addition, neither is division to subtraction.

Sides are distinct from roots, in the same way as lines are distinct from surfaces.

What comes out of an <operation> of division is called a parabola.

What numbers a particle or particles and is written above is called numerator.

What defines a particle or particles and is written below is called denominator, or name, or annotator.

The main species of ratios are the following: equal, inverse, mutated, composite, divided, ordinated and perturbated.

The square is the number which is obtained from two equal numbers.

The cube <is obtained> from three.  Such equal numbers are defined as the sides.

## Assumptions

## Sec. IV

We can take numbers equal to a number or multiples of it, greater or smaller.

The series of numbers can be produced infinitely.

No number can be diminished infinitely.

## Propositions

Sec. V

Various denominators are reduced to one and the same, once the numerator of a first fraction is multiplied by the denominator of the second, and inversely the second numerator by the first denominator, finally the denominators by each other.

Addition, subtraction, and division of particles, after having reduced the denominators, are the same thing. Subtraction of fractions is exactly the division of particles.

The multiplication of particles is obtained by multiplying the numerators and the denominators by each other: this is the sum of ratios.

A rule of proportions gives the fourth term, by multiplying the second term with the third and then dividing it by the first. In the same way it is possible to obtain the remaining ones, multiplying the others separately, provided that the first and the third are always of the same genus.

The canon of the simple hypothesis follows the rule of proportions, when a certain number is taken, and the reasoning of the question is concluded from it.[189]

Theorem for the use of the canon of the double hypothesis

If instead of an unknown number of a question any two numbers are

189 See ***De Arte magna***, Book 1. As we have stressed in Chapter 3, this part of arithmetic is the rule of false position which is used also in XVIth century algebra. Gosselin gives it, and the following, a great deal of space in his algebraic text. Then, he gives a similar definition to the *fictitia aequatio*.

taken, and if the formula of the question is completely treated from both sides separately, and if something then remains or is lacking, let it be indicated by the symbol of greater or smaller, the difference of the action of the errors with respect to any one of the errors themselves, will be the same as the difference between the hypotheses with respect to the error of the assumption of the second, which is assumed in a proportional way.[190]

The error in the hypothesis will produce the required number, when added to the hypothesis, if the hypothesis is smaller and it is appropriate or when subtracted, if greater.

From this it follows that both genera of hypothesis elicit the true from the false.[191]

## Theorem on the proposition IV of the II Book of Euclid applied to numbers, and made general, for the extraction of the square root.

If we divide a number into any parts, the square of the whole will be equal to the square of the parts, plus the double of the <product> made

190 This procedure of determination of a value by assuming another value involves a computation of errors to which Gosselin devotes great attention also in his algebra.

191 This is, in fact, what Gosselin also gives as a definition of algebra.

from each of the parts with the remaining parts.[192]

Theorem proportional to the previous one

for the extraction of the cubic root.

If we divide a number into any number of parts, the cube of the number of the whole will be equal to the cube of the parts, plus the triple of the product of the square of a part by the other, which is as true as it is difficult to understand, if you do not pay attention.

There is one way to do multiplication and division of fractions, [proceeding in opposite ways] one contrary to the other. For, if the multiplication takes place by two, the terms are multiplied by squares, if by three by cubes, and so on. If the division is by two, the square root <side> of the terms is extracted. If by three, the cubic, if by four the fourth, and so on infinitely.

## Cap IIII

## Definitions on the more subtle arithmetic.

### Sec. I

192 Both this and the following section describe the products commonly used in algebraic calculations. Gosselin selects the topics useful for algebra.

It is peculiar that in the square and cubic names, with which the rougher arithmetic concludes, there operates a subtler[193] arithmetic, as in simple numbers. For example one does not distinguish squares from four except through the quaternary; or similarly, the cube from three through the ternary; and what makes the limits of rougher arithmetic constitutes the principles of the subtler one. Such principles are almost infinite, and the main ones are the Side, the Square and the Cube.

The goal of science is the knowledge of the unknown quantities, and the means to that end is the equation, or equality.[194]

The number is a multitude made of monads alone; and there are four species: absolute, particular, rational and without ratio.

The name is a term in a continuous geometric proportion, established by the order of the eighth proposition of the ninth book of Euclid.[195]

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193 This term was commonly used, also at the time, to indicate the difficulties of mathematics or of mathematical texts. *Ton subtil Diophante* is used by Courtin de Cissé, while Xylander calls the problem II, 8, later made famous by Fermat "*praeclarum problema et rarae subtilitatis*." Here, however, Gosselin seems to be inspired by Nunez. In his dedicatory epistle he writes "*E posto que os principios desta subtilissima arte sejam tirados dos Liuros elementarios de Euclides, nam se pode porem sem ella ter a practica dos mesmos liuros, & dos de Archimedes*." (***Libro de algebra***, ii) Notice that Gosselin never uses, in this text, the word "algebra." This term had been questioned for its "impurity", already by Jean Borrel, who replaced it by *Logistica* to describe the discipline, and by *quadratura* to describe the solution of equations.

194 It is interesting to compare this definition with the one given in ***De arte magna***, or by other contemporary authors. This time the stress is on equation. E.g., Nunez writes: "(...) *Algebra, que he conta facil & breue para conhecer a quantidade ignota, em qualquer proposito de Arithmetica & Geoemtria, & en toda outra arte que vsa de conta & de medida*."(ibidem)

195 This is the common justification for the sequence of powers of the unknown; it is present, for instance, in Stifel.

The value of the name is the series of terms.

A particle or particles occur when both numbers, or only the denominator, are indicated by a name.

An equation occurs when some quantities of different genera are revealed to be equal to each other.

The equation is simple when there are only two <members>

Composite, when there are many to one, or many to many.

It is equatitious,[196] when the equation comes from the necessity of the operation.

Fictitious,[197] when hypotheses are based on the arbitrary decision of the arithmetician.

The equation is said to be duplicated,[198] when the reduction to a simple equation of two equations <is performed> through quantities which multiply the interval[199] of the numbers.

An equation is corrected, when the terms of the equation, which were without ratios, are reduced to the rational, after the other hypotheses have been substituted.

An equation is said to be intermediate if while correcting the equation one has to investigate a name between two numbers.

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196 This notion is obviously defined in opposition to the next.

197 This notion, as the following, comes from Diophantus; the specific terms are from Xylander's translation, problem II, 9. See ***De Arte magna***.

198 Similarly, see Diophantus' ***Arithmetica***, II, 11. In Xylander's version, II, 12.

199 I.e., which are factors of the common difference in an arithmetic progression.

Adequation is said to occur when, in finding the names to explain the problem, we apply our hypotheses to a side close to a certain given number, as if it were the right one.[200]

Assumptions

Sec. III

Let us assume that if we have to subtract a quantity from another when the names are different, written next to the other with the minus sign, it is understood as subtracted. Let the sign of lack be -.

A quantity that does not bear the sign of lack, is understood to have the sign of abundance: let its sign be +.

The sign + subtracted from the sign - and the sign - subtracted from the sign + leave <their remainder> with the sign of the excess.

Equal signs multiplied or divided by each other give a sign of abundance.

Instead, different signs give a sign of lack.

Furthermore, quantities of a different name are added or subtracted by

200 This is Xylander's translation of *parisotes*, the term used by Diophantus in Book V, prop. 14 (11 in modern editions). Here the definition is explicit, and is obviously connected to that of the rule of false position.

the signs + and -.[201]

Propositions

Sec. IV

Given a name, different names are multiplied reciprocally according to the sum of the values: but they are divided according to the difference.

The sides of a different name are multiplied, by multiplying the numbers among themselves after they have been first called the same name and by taking the side of the product; they are also divided, after dividing the numbers among themselves, and taking the side of the parabola.

The sides of a different name are added, first by reducing to the same name, then by dividing the greater number by the smaller, by adding one monad to the side of the common parabola, finally by multiplying the sum by the smaller side.  One subtracts, after having subtracted a monad from the side of the parabola, by multiplying the residual by the smaller side.

To get a simple equation, we shall divide the value of the quantity which has a lesser denominator by the number of the quantity of the greater denominator, and we shall subtract the smaller number from the larger. And this second residual denominator will be explained as the parabola of numbers: as the required quantity.

In developing a fictitious equation, the required name will be formed by

201 As we have seen, Gosselin does not use these signs in his ***De Arte magna***.

the root, by another number of numbers, plus or minus some monads: or some monads plus or minus some numbers, with the proviso that until a species is applied to a species of different genus, the equation will stay simple.

To treat the double equation, having examined the interval between two equations we shall find two numbers which, in their multiplication, will give that interval. Of these, either the square of half the interval is equal to the quantities of the smaller equation, or the square of half the sum is equal to the quantities of the greater equation. It is fitting however that in such a way the numbers which multiply that interval be made explicit, so that they are included between the limits of the simple equation.

The first canon of the composite equation when the number is equal to the squares and to the sides is of this type: to multiply the number by the monads of the squares, to that we add the square of the sides of half the number, and we subtract from the side of this sum a half of the number of the sides and we divide the remainder by the monads of the squares. The parabola will be the required side.

The second canon of a composite equation, if the number and the sides are equal to the squares, is defined in the following way: that we add to the number multiplied by the monad of the squares the square of half the number and the sum of the sides, then we divide this sum by the monads of the squares: the parabola will be required side.

The third canon of the composite equation is this, when the sides are equal to the squares and the number. Then we shall subtract the number made from the number in the monads of the squares from the square of half the number of the sides, we shall add the square side of the remainder or we shall subtract from half the number of the sides; the sum or the remainder divided by the monads of the squares shows the required side.

The fifth proposition of Book II of Euclid and the thirteenth of Book I of Diophantus confirm with their demonstrations the three propositions above, and those which are proportional to them.[202]

## The arithmetical ternary, and the propositions still wanted.

### Sec. V

To determine the values for the equations which are between the Sides, the Cubes and the number; the Squares, the Cubes and the number; the Squares, the Sides, the Cubes and the number.

These are the things, gentlemen, with which we have amused ourselves. Although we have neglected many propositions here in mathematics, a good part of it, even, if I am not wrong, all of it, is with us. And it is fitting not to display the greatest strengths of mind in these elementary works.

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202 I.e. biquadratic equations.

In fact, my mathematical energies are engaged now in the thirteen books of Diophantus of the Vatican.

I see your colleague Viète, as well as Cujas and Holler most renowned senators, and noble mathematicians, as full of expectations for the diophantian mathematics. In fact, men loved by all Muses, you believed that <Diophantus> the most erudite of the Greeks in Mathematics, full of mistakes, can be corrected by Gosselin. It is unlawful for me to deceive your opinion of me, so, hopefully, at the proper time I shall devote my energies to it.

FINIS

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# **Chapter Five: Mathematics and Rhetoric**

## *1. Introduction*

We have already indicated how the shaping of algebraic treatises was an important part of Peletier's program, and how this aim was brought forward by Gosselin. In this chapter we shall see more precisely the theoretical framework for writing algebra according to rhetorical rules, as well as the actual *structure* given by these two authors to their algebraic manuals. Furthermore, we shall see how this self-conscious new style in mathematics was parallel to an increasing interest in the transformation of rhetoric into a linguistic tool for the construction of a rigorous discipline.

We shall recall the more general context in which treatises and teaching manuals of algebra were written, in order to see how rhetoric operated in the constitution of the algebraic text by Peletier, Gosselin, Viète, and in the schema of Descartes, both as an implicit discipline of writing, as well as a strategy for constituting scientific discourse.

Algebraic books, from the time of the abacus schools, traditionally conclude with the treatment of a series of problems or questions. Thus, we shall interpret these final sections in the light of the rhetorical theory of the *quaestio*, which constituted, at the time, a topic of reflection and debate. We shall conclude with a discussion of the identification, proposed by Viète and some of his contemporaries, between algebra and the logic of scientific discourse.

## *2. Peletier's text L'Algèbre*

Peletier could not have been more explicit in stating his rhetorical projects concerning algebra: he thought that he had contributed to "quelque partie de l'invention et presque toute la Disposition"[203] (end of the *proème*, f. 8). We shall see whether he in fact

203 Jacques Peletier du Mans. ***L'Algèbre***. Lyon, Jean de Tournes, 1553. I shall quote here the copy belonging to the réserve of the Bibliothèque Nationale de Paris, Rés. V. 2074.

lived up to this statement.

In chapter 2 and 3 we saw that Peletier had two main models: the ancient *algorismus* and recent algebraic texts. The *algorismus* constituted the structure of the medieval arithmetical manual which introduced Arabic numbers in the West.[204] It was structured around the four operations, as these were explained first for integers, then for decimals, fractionals, and finally for sexagesimal numbers. This structure established an order of increasing complexity towards a gradual mastery of computation.

In the tradition of the abacus schools in the fifteenth century, this structure was expanded to include some rules of commercial arithmetic, and some algebra. The first printed books dealing with algebra followed this scheme, as did the two main sources for Peletier, Cardano (***Practica arithmeticae***, Milano, 1539, ***Ars magna***, Nürnberg, 1545) et Michael Stifel (***Arithmetica integra***, Nürnberg, 1543). These authors had in fact introduced cossic numbers, algebraic numbers, or terms containing unknowns and with signs, only at the end of the complete *algorismus*. In this way, these authors had shown that they considered cossic numbers the crowning achievement of the traditional *algorismus*.

Furthermore, Stifel added at the end a treatment of irrational numbers according to the tenth book of Euclid's ***Elements***, as well as of cossic numbers, and he gave his work the title of ***Arithmetica integra***, to stress that his arithmetic included all kinds of numbers. By contrast, the first algebraic text published in France,[205] the work of the Tübingen professor

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204 See, for instance, the essay by Gillian R. Evans, "From Abacus to Algorism: Theory and Practice in Medieval Arithmetic", ***The British Journal for the History of Science***, Vol. 10 No. 35 (1977).

205 One should mention Etienne de La Roche. ***L'Arithmétique***. Lyon, C. Fradin, 1520, as well as Gemma Frisius. ***Arithmetiae practicae methodus facilis***. Anvers, G. Bontius, 1540 et G. Richard, 1545, which contain some algebra in a much larger context of commercial arithmetic.

Johann Scheubel,[206] was very short, but built directly on a new principle, i.e. a simple classification of equations. Here, as in later algebraic treatises, the order of complexity was represented by the degree of equations.

Peletier chose an intermediary solution:[207] his work is in two books, the first of which is devoted to rational numbers, and the second to irrational numbers. The complexity is therefore still seen in terms of numbers and numerical solutions, and not of equations, degrees or solution formulas. Here Peletier follows Stifel. Like Stifel, in the course of the first book, he deals in a unified manner with the various solutions of the second degree by a single procedure. On the other hand, he differs from Stifel and comes closer to Scheubel insofar as he starts immediately, at the beginning of the volume, with the definition of algebra and of cossic numbers: his work is a true algebraic treatise. Furthermore, he develops at length a theory about the notion of the equation and the solution formula for second degree equations found by Stifel, but even more than Stifel, he stresses the generality of this formula.

We have seen in a previous chapter to what extent Peletier made use of Cardano's manuals. However, stylistically, Peletier did not borrow from Cardano. In fact, as we shall see, he deliberately set out to modify Cardano's style.

Taking off from Scheubel's compendium and Stifel's arithmetic, and making use of Cardano's mathematics, Peletier composed a full-fledged algebraic text, like Cardano' ***Ars Magna,*** entirely devoted to algebra, and comparable to it also in terms of completeness. A

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206 Johann Scheubel. ***Algebra compendiosa facilisque descriptio***. Paris, G.Cavellat, 1552.

207 Peletier's algebra is described by H. Bosmans, "L'algèbre de J. Peletier du Mans", in ***Revue des questions scientifiques***, 61, 1907, 117-173, and, some years later, and in less detailed way, by M. Thureau "J. Peletier, mathématicien manceau au XVIe siècle", in ***La province du Maine***, 2nd série, 15, 1935, 149-160, 187-199.

further innovation, relevant to Peletier's doctrine, was the decision to write in French, instead of Latin. Peletier could therefore rightfully consider himself an innovator in the field of the *disposition* of algebra. What is relevant here is his use of the technical terms of rhetoric to explain his elaboration of algebra. Furthermore, he articulated in this way the theory of dialectic which we are accustomed to attribute to Ramus. We find a relevant statement on this point in the *proème* of the first book of his ***L'Algèbre***:

> En tout ouvrages, qu'y a-t-il que l'ouvrier se puisse dument approprier, si ce n'est la forme? Il n'y a rien en l'oraison qui soit de l'Orateur, si ce n'est ce qu'on appelle la collocation. Car les mots, ni même les sentances, ne sont point du sien. Les mots, sont du Peuple, les sentences, sont des conceptions universelles des Philosophes. Quelle louange appartient-il à un homme pour entendre ni pour parler une langue, s'il ne sait accomoder les mots, et les accoutrer artificiellement à son point et à son besoin? Comment les accommodera-t-il, sinon avec jugement? En quoi git le jugement, sinon en l'ordonnance? (...) Si la Disposition est celle qui donne dignité aux choses, et si la forme est celle qui fait être une chose celle qu'elle est, je me promet de m'être ici tellement acquitté.

We have seen in Chapter 1 how Peletier was involved in the literary debate of the time. Here he evokes the classical situation of the Orator, so that writing can be described in terms of rhetoric. We know that Ramus, during the same years, proposed a reform of dialectic and that, in particular, according to Ramus invention and disposition belonged to dialectic and not to rhetoric.[208] Ramus, like Peletier, put disposition in the foreground, and on this fact depended his particular notion of method, privileged with respect to invention. However, invention precedes method in the order of knowledge. This theory corresponded, at least in part, to his practice of writing teaching manuals, inasmuch as both Ramus' ***Gramere*** and his

208 See the classical works on Ramus and his method, such as Walter Ong ***Ramus, Method and the Decay of Dialogue***, Cambridge, Harvard U.P., 1958. J. J. Verdonk, ***Petrus Ramus en de wiskunde***, Assen, 1966, C. Vasoli ***La dialettica e la retorica dell'Umanesimo. "Invenzione" e "Metodo" nella cultura del XV e XVI secolo***, Milano, Feltrinelli, 1968. See also the more recent Nelly Bruyère ***Méthode et dialectique dans l'oeuvre de La Ramée. Renaissance et âge classique***. Paris, Vrin, 1984.

mathematical text follow the order from the universal to the particular. More generally, these were for Ramus the foundations of a ***mathesis universalis***.[209] In fact, M. Magnard[210] has shown the connection between this "structural" view of logic and ontology and Ramus' theory of grammar, appropriate for uninflected languages. We might ask ourselves if Peletier here is following Ramus' point of view (Peletier's text is posterior to the two editions of Ramus' ***Aristotelicae animadversiones***, of 1543 and of 1548), or if he is working on his own in a similar conceptual framework. The two authors certainly have this in common, that method (or disposition) was not only a tool for the orator, but also a moment in the process of knowledge. On the other hand, it is plausible that Peletier had his own theory on this topic. This is not the only case in which he has priority over Ramus. Peletier had been the first to theorize the need to write mathematical texts in French, a position later taken by Ramus, and he developed a theory of orthography which Ramus would quote in his ***Gramere***.[211]

In order to clarify what Peletier meant by *disposition* in algebra, we can recall two passages in which he seems to make it explicit.

In the first passage, which occurs in the ***Algèbre,*** he mentions the authors who preceded him, and stresses their limits:

> De Cardan je dirais qu'il a enrichi l'algèbre de belles inventions, avec Démonstrations laborieusement cherchées, mais un peu confusément, et très

209 See G. Crapulli ***Mathesis universalis. Genesi di un'idea nel XVI secolo*** Roma, Edizioni dell'Ateneo, 1969.

210 See P, Magnard "***L'enjeu philosophique d'une grammaire***", in "Pierre de la Ramée", ***Revue des sciences philosophiques et théologiques***, 1986, 1.

211 Peletier also has priority over Ramus with respect to linguistic ciceronianism and the doctrine of oratory. This which is treated in detail by Kees Meerhoff ***Peletier, Ramus et les autres*** Brill, 1986.

> obscurément. De Stifel, je dirais que bien il a mis toute la peine qu'il a pu, de reduire l'art en sa simplicité: et en cela, a plus fait que nul autre auparavant lui. Mais il a un peu trop amplement parlé es endroits faciles, et trop chichement es difficiles. En somme, je dirais de tous ensemble, qu'ils ont eu peu d'égard à la méthode et ordonnance.

Peletier's goal is therefore made clear by contrast with his two main sources. He contributed to algebra precisely what the previous algebrists did not possess, i.e. a sophisticated rhetorical theory and the skills adequate to apply it in practice. Method was the feature that would distinguish his teaching manual from the previous algebraic tradition. In this way, Peletier proceeded to transform his art into a science and a discipline.

The passage stresses the inadequacies of previous manuals in giving demonstrations. To simplify demonstrations was in fact one of the goals in Peletier's writing, in particular with respect to Cardano's algebra. In this sense, the method as rhetoric (the disposition) and the method as dialectic (the demonstration) converge. However, we should not speak here of a new logic or theory of demonstration, but rather of a rhetoric of demonstration. Peletier seems to suggest that thinking itself has a linguistic structure which can be made explicit by rhetoric, which is its specific theory.

To be sure, Peletier was aware of a question prominent in sixteenth-century discussions on demonstration, which was the attempt to find a way to reconcile Aristotle's and Euclid's logics. However, he responded to it in a way which mostly reflects his all-encompassing notion of rhetoric. We can see this point in the first chapter of Peletier's edition of the first six books of Euclid's ***Elements***, a text published many times, from 1557 up to the beginning of the seventeenth century:[212]

> La Démonstration les Dialecticiens l'appellent le syllogisme qui fasse savoir: à savoir, qui des choses fort prouvées fait sa conclusion. Et cette

212 In fact, this text belongs to Jean II de Tournes' French translation of Peletier's work.

> démonstration prend son origine de la Géométrie. Qui plus est, toute preuve, qui nous mène à la vérité, est Géométrique. Tellement qu'il a été dit, que nul ne saurait distinguer le vrai d'avec le faux, s'il n'est bien versé en Euclide.

So far, Peletier simply repeats some well-known ideas. However, he explicitly stresses the fact that demonstration is a kind of syllogism. This point appears even clearer in what follows:

> Que si quelqu'un recherche curieusement, pourquoi en la démonstration des propositions ne se fait voir la forme du syllogisme, mais seulement y apparoissent quelques membres concis du syllogisme, que celui là sache, que ce seroit contre la dignité de la science, si quand on la traite à bon escient, il falloit suivre ric à ric les formules observées aux écoles. Car l'Advocat, quand il va au barreau, il ne met pas sur ses doigts ce que le Professeur en Rhétorique lui a dicté: mais il s'étudie tant qu'il peut, encore qu'il soit fort bien recours des preceptes de Rhétorique, de faire entendre qu'il ne pense rien moins qu'à la rhétorique.

Thus, not only is mathematical demonstration identified with a form of syllogism - as we noted above - being for instance a syllogism of the first figure, but by this analogy syllogism is conceived of as logical rule *used* in mathematical demonstration in the same way rhetoric provides the rules for a good speech. Even though expressed as a simile, this analogy compares logical and rhetorical rules, in spite of the fact that according to the usual definitions, especially of those of Aristotelian origin, rhetoric concerned only probable and non-rigorous demonstrations. Incidentally, we should keep this analogy in mind, because the reference to the world of the Palais de Justice as a place of expression of "scientific" rhetoric would become an ideal in the following generation. And Peletier concludes:

> Ainsi, en l'oeuvre géométrique, veu que nous ne cherchons rien autre sinon d'atteindre justement au but que nous désirons nous dissimulons entièrement la figure du syllogisme. Laquelle toutefois si on voulait rechercher, elle se pourroit exprimer au vif des preuves Géométriques.

On this specific theoretical point, Peletier's and Ramus' positions differ. They both maintained that Euclid had to be translated according to new criteria. But according to

Ramus this meant making *analysis* explicit, rather than the synthesis provided by Euclidean demostrations. From Ramus' point of view, there was no relation between syllogisms and Euclidean demonstrations. In fact there was no Aristotelian demonstration which could be as certain as mathematical demonstration, whereas *mathesis* was the best way to refound logic to make it work through an inductive process. Peletier had a different point of view, even if when he writes that *Toute preuve, qui nous meine à la vérité, est Geometrique* he seems to be taking a similar line. However, he also specified that the syllogism is the intrinsic logic of mathematical arguments; it is the logic they *use*.

This passage is the best illustration of what a different point of view in rhetoric can mean in mathematics. In technical terms, we notice the distinction between Cicero's thesis, according to which the *ordo* is in the invention, and Horace's thesis, according to which the *ordo* is in the *elocutio*, that is to say, in discourse proper. The former is represented by Ramus, the latter by Peletier. Both authors had a discursive notion of thought, so that logic should be conceived as rhetoric or *ratio disserendi*. Both followed Cicero[213] in seeing it as divided into two parts, *inventio* and *iudicium*. Certainly Ramus, and probably also Peletier, denied that there was a radical distinction between apodeictic and dialectic arguments. But they saw mathematical arguments differently. Peletier saw them as reducible to syllogisms, whereas Ramus denied the possibility of this reduction. Peletier had written this passage[214]

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213 See ***Topica***, II, 6: Cum omnis ratio diligens disserendi duas habeat partis, unam inveniendi, alteram iudicandi, utriusque princeps, ut mihi quidem videtur, Aristoteles fuit. Stoici autem in altera elaboraverunt. Iudicandi enim vias diligenter persecuti sunt, ea scientia quam *dialectiken* appellant. Inveniendi vero artem, quae *topike* dicitur, quae ad usum potior erat et ordine naturae certe prior, totam reliquerunt. Nos autem, quoniam in utraque summa utilitas est, et utramque, si erit otium, persequi cogitamus, ab ea, quae prior est, ordiemur. (Ed. H. Bornecque)

214 Peletier's "Euclid" was published in 1557.

about fifteen years before Ramus' polemic against Schegk on mathematical arguments.[215] While we might tend to think that the precise relation between the different kinds of argument was simply unproblematic for him, we should also recall that Piccolomini's discussion on mathematical proofs dated already to 1547.[216] Hence, we can consider his statement as the definition of a different view. In origin, I think the difference should be interpreted as follows. For Peletier the rules of rhetoric and of logic should be seen as the skeleton of scientific discourse, whereas for Ramus they constituted the point of departure of the *theory* of science as distinguished from its *discourse*. This corresponds of course with what the two authors actually produced, Ramus giving schemes of treatises, and Peletier publishing some of the most innovative and well diffused mathematical treatises.

For, Ramus excluded the identification between Aristotle's logic and Euclidian demonstrations, as he stressed in his dispute with Jakob Schegk.[217] Elsewhere, Ramus criticised Euclid because of his *method* of demonstration, on the grounds that synthesis obscured mathematical reasoning. Here, what is common to both authors is their sensitivity to a new presentation of mathematics. Ramus motivated this change in connection with the pedagogical reform at the university of Paris. Peletier motivated it in connection with the writing of teaching manuals appropriate to the new public offered by the printing press.

In fact, already in his ***Arithmétique*** (1549), Peletier had dealt with the question of obscurity in writing. He proposed *clarté* as the quality which, together with *brièveté*, should mostly inspire writers, particularly at the age of the printing press:

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215 On this polemic, see C. Vasoli, ***La dialettica e la retorica...***

216 ***Commentarius de certitudine mathematicarum...***

217 See the details of this dispute in Vasoli 1968.

> Entre les hommes d'érudition, a été longuement debattu, et n'est encore le différend vidé, lequel des deux est le plus profitable pour l'entretenement des disciplines, que les professeurs d'icelles, quand ils les mettent par écrit, les traitent clairement et au long, ou bien obscurement et brief.

After having taken into consideration the various alternatives and their advantages in teaching, Peletier concludes:

> A la verité nous voyons qu'aujourd'hui on a trouvé moien d'abréger le temps aux disciplines par clarté et facile manière d'enseigner.

And below:

> J'ai pris opinion de suivre un chemin mitoien. Car après avoir bien examiné le mérite des deux contraires, je trouve qu'il n'est pas impossible d'être facile et brief tout ensemble, pourvu qu'on tiegne toujours son adresse à la méthode, qui est celle qui donne majesté aux écrits, et non l'obscurité.

Here, the use of rhetorical and/or sociological themes by Peletier calls to mind Peletier's broad project (discussed in chapter 1) for the development of a national language, to be enriched and elevated by mathematics. In particular, we find Peletier's idea of *clarté* and *brièveté* which he made fully explicit in ***L'Art poétique***: "la première et plus digne vertu du poème est la clarté." It is in this context that we can interpret some surprising statements by Peletier. For, he seems to be the first to attribute to algebra "la partie la plus occulte des mathématiques", a kind of science of sciences. Peletier writes at the beginning of the first chapter of his ***Algèbre***:

> L'algèbre est un art de parfaitement et précisément nombrer: et de soudre toutes questions Arithmétiques et Géométriques de possible solution par nombres Rationnaux et Irrationnaux. La grande singularité d'elle, consiste en l'invention de toutes sortes de lignes et superfices, où l'aide des nombres rationnaux nous défaut. Elle apprend à discourir, et a chercher tous les points nécessaires pour résoudre une difficulté: et montre qu'il n'est chose tant ardue, à laquelle l'esprit ne puisse atteindre, avisant bien les moyens qui y adressent.

Algebra can teach how to *discourir*, which is, for Peletier, synonymous with *raisonner*. This is a notion that appears here for the first time, even though it is an idea that occurs frequently in seventeenth-century authors. We should also notice Peletier's idea that algebra teaches how to see all relevant points to solve a problem. Finally, we recognize, in this new "methodological" context, the idea that algebra allows people to solve all problems, a *topos* which belongs to the medieval tradition of algebra, but is well known in Viète's formulation "nullum non problema solvere." Here, this motto is associated to two other aspects, on the one hand, the possibility of giving a good formulation of a problem, through algebra; and on the other, the capability of extending the universe of problems.

This *topos* was not, for Peletier, only a passing remark. As will be thoroughly explained in the Appendix, in connection with Descartes' ***Regulae***, Peletier was the first to make explicit a point merely sketched out by Cardano. In fact, Cardano had included in his ***Practica arithmeticae*** many *examples* (which were also called questions or problems) taken not only from the commercial world, as was customary, but also from natural philosophy, and from geometry.

Peletier developed this point in his ***Algèbre***:

> L'algèbre, pour sa perfeccion, presuppose la cognoissance de toutes sortes de Teoremes, comme de Geometrie, d'Astronomie, de musique, de Phisique, e brief de tous ars e sciances. [p.113]

Such was the interaction between algebra and rhetoric in the first phase of the French algebraic tradition.

### *3. The algebra of Guillaume Gosselin*

Turning now to the main instance from the second phase of the French algebraic tradition, Guillaume Gosselin's ***De Arte Magna***,[218] let us examine its structure as we have done for Peletier's text.

Gosselin was the second author of an algebraic text to deal with Diophantus'***Arithmetic*** in his work, the first being Rafael Bombelli in 1572. The sources he mentions at the beginning of the book, besides Diophantus, are Pacioli, de La Roche, Forcadel, Cardano, Stifel, Scheubel, Peletier, Tartaglia, Borrel (Buteo), and Nunez.

For *dispositio* of his text, Gosselin is inspired by two works, first, Peletier's ***L'Algèbre*** and then Ramus' ***Algebra***.[219] The latter contained an explicit indication according to which, in teaching algebra, it is necessary to start from the numeration (*algorismus*), then to go on to proportion (computation on proportions, following the tradition of Eudoxus and Euclid), and finally to conclude with the treatment of equations. We should add, in passing, that Ramus' treatise was very brief and not otherwise innovative.

Gosselin's treatise includes a first book devoted to the *algorismus* of *quantitates*, i.e. of the algebraic numbers. It deals with operations with rational and irrational numbers and with the extraction of roots; a second book is devoted to proportion; a third book, defined as the most important, is devoted to the different sorts of equations; finally the last one deals with equations in several unknowns. What strikes the reader is the exclusion of practical problems, which were still present not only in Peletier's text, but also in Borrel's book.[220] All

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218 Guillaume Gosselin. ***De arte magna***. Paris, Gilles Beys, 1577. Henceforth, I shall cite the copy at the Bibliothèque Nationale de Paris, V. 20151. H. Bosmans has devoted a long article to Gosselin's algebra.

219 [Anonymous] ***Algebra*** Parisiis, A. Wechel, 1560.

220 Nunez, however, was also dealing with strictly "mathematical" problems only: arithmetical or geometrical.

problems, or questions, are numerical or geometrical. Numerical problems are treated in the style of Diophantus' problems, both in the section concerning the *algorisme* and in the theory of equations. Geometrical problems are treated according to the model of Euclid, as reinterpreted by Borrel, Tartaglia and Nunez, who demonstrate the solution formulas in a geometrical manner.

Gosselin's innovation consists mainly in the use of Diophantus' ***Arithmetic***. This innovation adds substantial new material to the previous theory of equations. The definition of equation and the need for a good formulation of a question in terms of the equation constitute in fact an important moment in Gosselin's theory. Also, the rediscovery of Diophantus allows Gosselin to extend the notion of problem. In this way, new sorts of problems are conceived of in a more general way. We see here the making of a theory to solve problems of problems. Peletier's project of solving all problems in mathematics,[221] with the sole condition of being able to correctly pose the equations, was thus realized for the first time in the field of algebra, and this allowed mathematicians to extend its applications to geometry, and to give algebra a main rôle among mathematical sciences.

In Gosselin's works, *quaestio* appears in many contexts. The most significant occurrence is in Gosselin's version of Tartaglia, ***L'arithmétique de Nicolas Tartaglia Brescian***, published in Paris in 1578. Here too, Gosselin gives algebra a place in the *quadrivium*. In his preface, dedicating the book to Marguerite de France, Gosselin mentions the many marvels made possible by mathematics, among which the most surprising is the possibility, offered by algebra, of solving all problems. He writes:

Ne semble-t-il pas estre une chose totalement repugnante à la nature, que de

221 See, above, on Cardano and Peletier and their idea of algebraic question as applying to all mathematical sciences.

> dissoudre toutes questions proposées, tant difficiles qu'elles soyent, [[& ce mesure d'une chose, qui ne peut estre, comme si elle pouvoit estre, et s'en servir generallement en toutes questions, & Problemes?]] entendre ce qui ne peut se faire, & ce que la Nature ne peut endurer, quelles choses sont toutes ces dignitez, qui passent le Solide, et toutefois par la vertu de ces Hypotheses & positions qui ne peuvent estre, venir finalement en la connaissance de ce qu'on demande? Cecy enseigne cette divine Algebre.

Thus, if on one hand the extension of the notion of problem or question depends on the development of mathematical techniques, it depends also, on the other hand, on the good *formulation* of the question, which gives rise to an equation. But the question was a topic of dialectic and of rhetoric, and particularly of Ciceronian rhetoric, as Ramus and Peletier had reconstructed it. To see this point more clearly, we must therefore focus on the notion of *quaestio*, at first by looking at Cicero's text itself, then by examining some examples of interpretations contemporary to our authors.

### *4. On the notion of quaestio, and Ciceronianism in Paris*

Cicero devotes to the *quaestio* a section of his ***Partitiones oratoriae***.[222] In fact, *doctrina dicendi* includes *vis oratoris*, *oratio* and *quaestio*. The *vis oratoris* is in things as well as in words, and includes invention and disposition, elocution, action and memory. The *oratio* is defined in its four parts: principium, narratio, confirmatio, peroratio. Finally, Cicero makes the *quaestio* the third part of the *doctrina dicendi*. He does not define it, but he establishes a distinction between the *quaestio finita*, which is defined as to time and as to people, and the *quaestio infinita*, i.e. the question which is not determined either with respect to time or with respect to people. The *quaestio finita* is called *causa*, the *quaestio*

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222 This is not an exclusive statement. More particularly, Cicero dealt with the *quaestio* in his related work *Rhetorica*. section 81. This work was also often edited by Parisians of and for instance by the Ramist *par excellence*, Omer Talon.

*infinita* is called *consultatio*. In the section which is devoted to it, Cicero gives the *quaestio infinita* the name of *propositum*, evidently connected to the aristotelian notion of *thesis*. Furthermore, Cicero indicates the two genres of *propositum* (or *quaestio infinita*), the first of which is *propositum cognitionis*, theoretical, and the second is *propositum actionis*, pratical. An exemple of the theoretical genre of *propositum* would be "can we trust the senses?", whereas an example of the genre *actionis* would be "by what menas can friendship be acquired?"

In this way it becomes clear that theoretical *proposita*, or *theseis*, are conceived of as questions, the answers to which can be scientific. In fact, Cicero analyzes this notion further, for he distinguishes three species: the *propositum* which studies the possibility of a thing (for instance: does the thing exist or not?), that which looks for the definition of a thing, for instance "whence comes virtue, from the nature of the will or from habit?", and finally that which tries to determine the quality of the thing. Cicero writes here:

> Cuius generis sunt omnes in quibus, ut in obscuris naturalibusque quaestionibus, causae rationesque rerum explicantur.
>
> [of this kind are all questions by means of which the causes and the reasons of things are explained, as in the case of the most obscure natural questions]

We see here, at least in general outline, how Cicero's rhetoric included scientific questions in his classification. What makes this classical context of an immediate importance for our purposes, is that Cicero's doctrine was at the center of rhetorical and philosophical debates in sixteenth-century Paris, as well as of the philosophical commitments of the French algebrists. In chapter 1 we have seen how Peletier's enthusiasm for the translation of mathematical books and the writing of algebraic treatises in French was in fact articulated in an interpretation of Cicero. Similarly, Ramus was also the author of a ***Ciceronianus*** which was intended to establish the canons of the *imitatio* in the vernacular.

In what I have called the second phase of the French algebraic tradition, the interest in Cicero was connected to two important aspects of the linguistic debate: the importance of the "langue du Palais", which was to become "elevated French", and, on the other hand, the revival of the use of Latin in printed works. These two aspects were not contradictory. In fact, they were supported by the same group of scholars belonging to the juridical milieu. Both aspects were hardly new: French had long been established at the Palais de Justice, and the Collèges had never stopped using Latin. But what was new was the self-consciousness of a large group of learned jurists, engaged on two fronts, simultaneously, taking up the humanist movement for national language, which fitted with the evolution of writing history and law, and developing the philological (linguistical and historical) skills which would allow them to "monopolize" the writing of Latin. In other words, high magistrates were aware of determining the most elaborate style of French, on one hand and, on the other hand, the features of Latin in printed works. We know that the milieu of the Parliament was crucial for algebra. For, algebra was finding an audience precisely in this milieu, and thus came to inspire a high magistrate such as François Viète, who adopted a style which made it into a discipline typical of this milieu.

In this cultural context, the various editions of Cicero's ***Partitiones oratoriae***, containing a great number of comments on the crucial passages, as well as some handwritten annotations which stem from their use in the Collèges, constitute a good reference for us insofar as it was a reference for the student, as the most basic doctrine of rhetoric and dialectic.[223] Suffice it to recall that a dozen editions of the ***Partitiones*** were published in Paris starting with the second half of the sixteenth century. In particular, one should remember that Giorgio Valla's commentary appeared in Paris a few times[224] together with the commentary of Omer Talon, Ramus' collaborator.[225]

I intend to give here some examples corroborating my hypothesis that there has been a contamination between the rhetorical notion of *quaestio* and the algebraic notion of quaestion or problem. The result of such a contamination is apparent in Descartes (see Appendix). While the algebraic notion of equation is so generalized that it can aspire to capture all sorts of scientific questions, the rhetorical notion of question is transformed so that it will be able to replace Aristotelian logic. However, there are some traces of this transformation in earlier authors and writings. In particular, the commentaries by two early Ramists seem to offer some threads of a connection.

Let us start with a professor at the University of Paris, Claude Mignault. He is known for having edited Alciati's ***Emblemi***. Already this text would be of interest for us, because it contains a long ***oratio*** on symbols, which includes some hints about what a humanist of the time meant by symbolic language. Another of his works important for us is

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223 I prefer to take for granted, here, the different definitions for rhetoric and dialectic for Peletier and Ramus

224 Many of these editions are present at the Bibliothèque nationale. See Bibliography.

225 M.***In Marci Tul. Cic. Paritiones oratorias annotationes collectae ex praelectionibus Audomari Talaei***. Paris, David, 1551. I worked mostly on a later edition, which I shall cite from: T. Ciceronis ***Partitiones oratoriae ad veterum codicum manu scriptorum exemplaria collatae, et innumeris mendis repurgatae, cum commentariis Iac. Strebei, Bartolomae Latomi, Christophori Hegendorphini, Ioannis fossani, Adriani Turnebi (qui adhuc inscriptus est Commentariis incerti authoris) postremo adiectis praelectionibus Audomari Talei***. Parisiis, Ex Officina Gabrielis Buonii, 1568.

his edition and commentary on Talon's Rhetorica, published by Gilles Beys.[226] What concerns us here is his edition, with extensive commentary, of the ***Partitiones oratoriae***.[227]

It is in fact much more than an edition, because Mignault adds to Cicero's text a series of commentaries which constitute autonomous pieces, the *syntagmata,* each with its own title. Thus, we find syntagmata devoted to topics closely connected to the algebra of the time: symbols, *notae*, *species* and the *quaestio*. Furthermore Mignault, according to the good Ramist tradition, accompanies the text by large tables, which develop the themes he considered the most important. In reading this text, one understands that the sections of the Partitiones which are relevant for us here, already at the time attracted attention and efforts at interpretation.

Mignault provides a crucial instance of the way in which the interpretation of Cicero can be adapted to Parisian climate. In fact, Mignault introduces an innovation into Cicero's text, by stating that the *vis oratoris* is the efficient cause, while the *oratio* and the *quaestio* are in the relation of *form* and *matter*. We read in the *tabula*:

> Quaestio, quae materia est sive subiectum orationis, eaque est vel infinita, quae et consultatio, propositum, graece *thesis* vel definita, quae et controversia, vel causa graece *hypothesis* cuius genera tria.(p. 12)

And later, in the text of the prooemium:

> Vis oratoris, nihil aliud est quam facultas, *dynamis* appellata Graecis, aut

226 Talon, Omer. *Rhetorica*. Comment par Claude Mignault. Paris:G. Beys and J. Richet, 1577.

227 I worked in particular on the fourth edition: M.T. Cicero. ***Partitiones oratoriae M. Tullii Ciceronis, et ad eas facili et aperta methodo complectendas, Tabulae et syntagmata, una cum Diatribis aliquot, quibus omnium praeceptorum vis, et usus oratoriae facultatis exprimitur, per Claudium Minoem Divisionensem Editio quarta a ceteris multo locupletior***. Francofurti, Apud haeredes Andreae Wecheli, 1584. The dedicatory epistle is, however, dated October 1575. Thus, I will cite from this edition.

> caussa efficiens orationis, quae tota est in animo,... quae posita est in contemplatione. (...) Duas partes alias, orationem et quaestionem, volunt esse, ut Formam et Materiam, quibus describitur Rhetorica illa quam practicam seu activam appellant.

We have seen in the initial definitions that the *vis oratoris* includes all the features of good oratory, from *inventio* to *memoria*. In Mignault's setting, they constitute a "mental" tool, but a universal one, both because it is intersubjective, in that it can be transmitted through teaching and learning, and because it can be applied to any matter. That matter is oration and question. Here, as with his other innovations, Mignault applies Ramus' theory.

In particular, here is the kernel of Ramus' idea on the relation between theory and practice, that there are no separate practical and theoretical disciplines, but only a universal *mathesis* which is **useful** for all disciplines, and can be used theoretically or practically. Mignault concludes by stating that there is only one rhetoric. So, the meaning of the previous distinction is made clear later:

> In Oratoria illa vi, generalia doctrinae huius praecepta describuntur, quae quidem primum sunt artifici meditanda et perdiscenda. At quae deinceps de Oratione, deque Quaestione traduntur, sunt regulae speciales, ut ad usum minore negocio generales illas priori hac parte comprehensas Orator referat. (p.17)

In other words, oration and question are the practical part in the sense that they require that *use* of the general doctrine of the *vis*.

The context here is the *coniunctio* of philosophy and eloquence[228] Ramus proposed after criticizing the old notion of their connection. In Ramus' formulation, which we find for instance in ***Rhetoricae distinctiones in Quintilianum***, Cicero and Quintilian should be criticized for having mixed rhetoric and philosophy in an unacceptable fashion: they had

228 Needless to say, this was a debate started much earlier in the context of Italian humanism. See Seigel on this topic, as well as Camporeale.

asserted that it was necessary for the orator to know philosophy and, in the same sense, the orator had to be *vir bonus*, that the truth and the strength (*vis*) of his language could only correspond to his knowledge and to his behavior. To this, Ramus answers that the *vis* is not only a talent, but also an art, in the mind, but common to all minds. As a consequence, when Ramus in turn, maintains the *coniunctio* between philosophy and eloquence,[229] he means that the orator's *vis* is a universal art, which is why he can apply it to philosophy. The art does not derive from his virtues or from his personal knowledge. The art is itself sufficient. From this proceeds Ramus' identification of some parts of rhetoric, i.e. *inventio* and *dispositio*, with dialectic.[230]

Similarly, by separating form from matter, Mignault can follow Ramus and propose the *coniunctio* of rhetoric with philosophy, asserting that rhetoric is *useful* for philosophy. The *vis oratoris* is, in this sense, the mental tool that can be applied in solving questions in natural philosophy. Thus the *coniunctio* rests on this distinction between form and matter of the *oratio*, while at the same time giving a central role to the *quaestio*.

The Ramist origin of this first statement by Mignault is confirmed by the fact that Omer Talon, in his commentary to the ***Partitiones***, takes the same turn, while making precise the connection with dialectic. In connection with the definition of *quaestio*,

> Tractatur hoc loco artis materia, quae profecto est eadem cum materia dialectici, sed tractatione differt. Nam quod appellatur a dialectici problema et protasis, (est autem haec dialectici materia) appellatur a rhetoribus quaestio. A quaestione oriuntur status et genera causarum, de quibus Cicero

229 For instance, this is the name he chose for his chair at the Collège Royal.

230 The best formulations of Ramus' theory on this point are to be found in Vasoli 1968 and L. Jardine 1974.

postea agit.(op. cit. p.155)

Here Talon defines the *quaestio* as the matter of the art of rhetoric, while stating that it is also the matter of dialectic. This is the identification between the dialectic used by philosophers and the dialectic used by orators which is typical of Ramus' conception. But the identification evidenced by the second statement, equates precisely that which a classical interpretation of Aristotle would have kept distinct -- the scientific and the dialectical problem.[231]

This view allows Mignault to specify Cicero's distinction between *quaestiones infinitae* and *quaestiones finitae*. To this passage Mignault devotes a *syntagma*,[232] in which he recalls the utility of the *quaestiones infinitae cognitionis* for the *causae*, which is the argument in favour of the art in the Ramist sense and its utility. Mignault also stresses the difference between the discourse of the philosopher and the discourse of the orator. The philosopher *concise et subtiliter*, the orator *ornate, splendide et populariter iisdem de rebus agit*.[233]

This caracterization of the two kinds of *quaestiones* corresponds to a cultural and social evolution taking place in late humanism. Even elementary teaching of the classics was oriented around the construction of a new social figure, the high magistrate who was, in sixteenth-century Paris, the incarnation of the humanistic ideal.[234] The *orator* and the *philosopher* could be the same person, insofar as the high magistrate would be called to

231 In the Appendix I specify this reading of Aristotle's text.

232 Il syntagma 29, page 128.

233 Ibid. p. 129. See also the use of *quaestio finita* and *infinita,* p. 131.

234 On the shift from the ideal of *vir bonus* to the ideal of *vir peritus*, see especially Grafton and Jardine.

pronounce *orationes* at the Court as well as to determine *questiones infinitae*, i.e. general juridical questions at the Palais. In this sense, the theoretical identification proposed by Ramus corresponded to a social function. But, on the other hand, the characterization also involved a distinction. The *orator* could not be identified with the *causidicus*, i.e. with the simple advocate dealing with a specific *caussa*. The *causidicus* was bound to a "violent" confrontation, whereas the *orator* was in fact pursuing a search for truth. The distinction in intellectual domain marked a social difference: on one hand, the great *orationes* of the magistrates and of the "avocats du roi", asking and arguing the *quaestiones infinitae et cognitionis*, the *theseis*, on the other hand, the "petits avocats", asking and arguing the *quaestiones finitae et actionis*, the *caussae*.[235]

It should be noticed that Mignault's statement makes use of a *topos*, present not only in later texts, but also in Peletier. In particular, we should compare it with Peletier's assertions in his ***Ars poetique***, contrasting the scope of the art of the poet and that of the art of the orator: the poet can talk generally and about any subject. whereas the orator must limit himself to special situations, to cases.

Already in his *Aristotelicae animadversiones*, Ramus had stressed the importance of the notion of *quaestio*.[236] However, if contemporary historiography in the history of scientific thought presents us with a sufficiently complex picture of the evolution of the doctrines of invention and disposition, there is still much to be done on the other aspects of rhetoric, especially on the *quaestio* which, being the matter of the oratio, must follow a

235 This opposition between the "petits avocats" and the Parlementaires (de Paris) is described as a *topos* for the jurists theorizing the novelty of the "rhétorique du Parlement", from Adrien Turnèbe to Antoine de Laval in Marc Fumaroli ***L'age de l'éloquence*** Genève, Droz, 1980, particularly at the pages 462-473.

236 See Bruyère ...

specific pattern. Two reasons, it seems to me, justify the fact that historiography has privileged the two tracks of invention and disposition, first because of the interest in the doctrine of Method, and second, because of the special attention paid to the work of Ramus. The two aspects are clearly intertwined, and the research has been fruitful. But I also think that from the point of view of the history of algebra, so crucial for later theoreticians of Method, *quaestio* has played a fundamental role because it has allowed consideration of the process of putting mathematical matters into the form of equations in a rhetorical mode. Secondly, it has been crucial because it has given a scientific conceptual frame within which there was a place for questions, i.e. for the practical, and yet essential, part of algebra.

So far, we have seen that Mignault conceived of a universal art of discourse and reasoning, and that it was expressed in rhetorical notions. In particular, the notion of *quaestio* was enriched by the new Ramist conception of dialectic. Finally, we have seen that Mignault proposed an ideal of the erudite public officer, knowledgeable not only in his discipline, but in any science, insofar as he possesses the key to all knowledge. Certainly these ideas have much in common with the ideal of algebra. But, most particularly, the notion of *quaestio* seems to be an important clue insofar as the term was present both in the rhetorical and in the algebraic context, precisely at the moment when the content of algebraic *quaestiones* was moving from specific to general, from commercial to "scientific." Thus, we can maintain that works like Mignault's commentary seem to have provided the framework within which to transform an art such as algebra, from the motivation to the terminology, and the expository structure.

It would be possible to multiply the examples of passages in which Mignault makes reference to dialectic and science, thereby confirming the intellectual connection with algebra. Let us recall only one. In his *syntagma 6*, *de Notatione*,[237] he enumerates the various uses of numeration and concludes by affirming that the *notae* allowed the study of nouns, the pursuit of deduction and thus, a contribution to science. These statements are confirmed by references to "Rodolphus" (Agricola) and to Aristotle, ***Posterior Analytics***, book 1.[238]

The spread of Ramus' ideas in teaching at the Parisian colleges in sixteenth-century France is well-documented. It is also clear that, aside from Ramus' ideas, interest in dialectic was quite typical of that milieu.[239] Here, I have attempted only to indicate the richness of a very common text. These well known theories were formulated in a way that has striking analogies with the notions and terminology associated with algebra. Most of all, these *vulgata* of dialectic show that the rhetorico-dialectical notion of *quaestio* was becoming even more important than in Ramus' texts, and that this can be related to the transformation of the notion of *quaestio* in algebra. In other words, by the end of the century, French algebrists could easily apply to their art the notions and the terminology learned so insistently and so early at the colleges. On the other hand, cultivated people educated in this way could appreciate algebra and promote it together with some grand myths about its generality and its potentialities. It appears that the milieu of the Parlement, in particular, was not only permeated by this interested in dialectic, but also ready to develop it in the direction of the *quaestiones infinitae*. In other words, if the rhetoric of the jurists and the *gens de robe* was useful for the algebrists, providing the style to transform it into a discipline, we

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237 See ***Oratoriae partitiones*** p. 36.

238 See ***Oratoriae partitiones*** p. 37.

239 See in particular Grafton 1981 and Blair 1990 for classical texts annotated in college classes.

have grounds to think that algebra could, in turn, appear useful to jurists, insofar as it could be seen as key for universal knowledge, and based on that linguistic competence which was supposed to be the feature of the perfect jurist.

We might add that without such optimism, symbolic algebra would not have acquired the preeminence that it had in the seventeenth century.

### *5. Viète's algebra*

Let us now briefly review several points in the work of Viète and Descartes, the most famous representatives of the French algebraic tradition. Here we will show how their conception of algebra included and developed the identification between algebra and rhetoric which we have already shown to be common to this tradition.

At first the patron of Gosselin, François Viète took on, after the death of the young algebrist who was to edit Diophantus, the task of composing an algebra which would follow the French tradition while making use of the richness of the mathematics of Diophantus, Pappus and Apollonius.

There is no need to emphasize Viète's use of rhetoric, since his ***Isagoge*** is a classic example of the humanism of the period.[240] Suffice it to say that it completes the process which we have followed in the history of manuals, introducing from the beginning calculation with cossic numbers, which he conceived as *species*, in order to develop in the second part the theory of equations. The proportions and equations are identified according

240 Among the texts dedicated to Viète, I limit myself to mentioning F. Ritter ***François Viète, inventeur de l'algèbre moderne*** Paris, 1895; J. Hofmann, introduction au facsimile des *Opera mathematica* de Viète (Frans van Schooten, Leyden 1646, Hildesheim 1970; J. Grisard François Viète, mathématicien de la fin du seizième siècle. Essai bio-bibliographique présenté en vue de l'obstention du doctorat du troisième cycle, E.P.H.E, Paris; K. Reich, H. Gericke ***François Viète: Einfürung in die neue Algebra*** München 1973.

to the famous formula which establishes the relation between equations and proportions:

Proportio potest dici constitutio aequalitatis; Aequalitas, resolutio proportionis.

Rather, we shall sketch how Viète articulates the relationship between rhetoric and algebra or, as he calls it, *analysis*.

Let us consider the famous first chapter of the ***In artem analyticen Isagoge*** of 1591.[241] The title of this first chapter is "La définition et la partition de l'analyse, et les choses qui contribuent à la Zététique." This passage is well-known in the history of the "method" because Viète defines the analytic art as the art of *bene inveniendi* in mathematics, thus establishing a fundamental relationship between algebra and rhetoric. Here I want to emphasize Viète's statement that Zetetic should be defined as the type of analysis which allows one to find an equality or proportion of the required magnitude with the given magnitude. Other types of analysis are for him to some extent subordinate, and have to do with the reduction of the complexity of problems and the use of already established theorems. But, concludes Viète, the object of Zetetic is established in the art of logic by syllogisms and enthymemata. Once again we find in a writer on algebra the relation between syllogisms and mathematical demonstrations not with reference to syllogisms of the first type, but with reference to probable syllogisms. Without dwelling on this connection between analysis and enthymema, it is enough to note that Viète ends the chapter by indicating the advantages of this algebra (or as he puts it, this logistic), which makes use of letters for both coefficients and known terms, over numerical logistic when it comes to formulating and resolving problems.

If from the point of view of logic strictly speaking, algebra only constitutes an

241 I have consulted the copy in the Bibliothèque Nationale, François Viète. ***In artem analyticen isagoge***. Tours, J. Mettayer, 1591. V. 1507.

analysis of a problem, without the synthesis, from the point of view of dialectic, algebra was entirely suited to the formulation of scientific propositions.

This programmatic introduction suggests that not only does algebra follow the law of dialectic, but that it *is* dialectic, and constitutes the model for scientific argument.

Viète's version of algebra was much admired but little known, because the criteria of Peletier for the writing of algebra, clarity and brevity were eclipsed in its pages, and would only reappear much later, when around the 1630's people begin to edit and comment on Viète's works, at the time of the generation of Descartes.

### *6. Descartes's algebra*

From antiquity on, the genre of writings known as *quaestiones* tended, at least formally, to represent a dialogue between teacher and student.

The notion of question in Descartes was rightly interpreted first of all with reference to this metaphysical and theological tradition. We may, however, wonder whether before Descartes, who gives a new technical meaning to this term, there was already a specific tradition making use of it in a similar manner. Were the *questiones* of the metaphysicians the prototype of question for Descartes or should we think, rather, of the section ***De quaestione*** of the ***Ars brevis*** by Lulle? We know that Descartes was familiar with both kinds of writing. I myself have tried to show the way in which this notion was used in the ***Regulae***.[242] As we have seen in the first chapter, we know something about the relationships between Peletier, Gosselin and Viète, we have very few documents testifying to Descartes' knowledge of the French algebraic tradition, and particularly of Peletier and Gosselin. In

242 See Appendix.

fact, the influence has been, most often, denied by historians. On the contrary, it was long thought that there was no French antecedent for Descartes' algebra, until Pierre Costabel[243] recently showed that Descartes in his ***De solidorum elementis*** combined the notations of Clavius and Peletier, thus giving evidence of direct filiation. This fact constitutes a basis for future researches.

The texts to which we should refer here are the ***Regulae***, the text appearing in the edition of Adam and Tannery under the title ***Calcul de Mons. Descartes (introduction à sa géométrie***), and the ***Géométrie***.

In the ***Regulae***, algebra is only sketched out, but we have in compensation an ample theory of *quaestio*, with its synonyms *problema* and *difficultas*, with their obvious relevance for algebra.[244]

As for the ***Calcul de M. Descartes***, it follows Peletier's order of exposition, with an algorism of rational algebraic numbers, an algorism of irrational algebraic numbers, a section devoted to equations and another to examples. The section on equations begins "Quand on veut resoudre quelques problèmes..." and introduces notation and the formulation of the equation. In this passage, the term question is used twice as a synonym of problem as it is in the examples.

And, of course, we have the classic pages of the beginning of the ***Géométrie***, in which the *questions* are identified with equations.

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243 Descartes, ***Exercices pour les éléments des solides. Essai en complément d'Euclide. Progymnasnata desolidorum elementis*** Ed. critique avec introduction, notes et commentaires par P. Costabel, Paris 1987.

244 See my discussion of the ***Regulae*** in the Appendix.

## *7. Conclusion*

We began by promising a reflection on the identification between algebra and scientific discourse. This identification is apparent in Descartes, but there are reasons to believe that this is only the end of a process already underway in the French algebraic tradition, both because this tradition makes early use of certain mathematical tools, and also because it establishes a precise relationship between rhetoric and algebra. In the initial phase of French algebra, from the time of Peletier and Ramus, rhetoric constituted above all a way of giving a form of classical discourse to an algebra which in fact arose from a tradition of non-latinists.

The initial Ciceronianism of Peletier and Ramus was followed by the Ciceronianism of the period of *grande eloquence* of the magistrates of Paris. In this milieu of humanist magistrates, represented for example by Antoine de Laval with his theory and practice of rhetoric, Marc Fumaroli has noted a re-awakening of Ramism and a new role for logic and mathematics. This period extends to the beginning of the seventeenth century, but I think above all of the group in contact with Paolo Manuzio. I believe that I have indicated the two main aspects of the relations established between the algebrists, and particularly Gosselin, and the milieu of the magistrates. Gosselin, who thus is part of this second phase of the French algebraic tradition found, like Peletier, in rhetoric his canons for the writing of manuals and in particular, found in the theory of the ***quaestio*** the tools necessary to conceptualize the algebraic procedures for the formulation of the equation, and thus to give to his theory the form of a classical discipline. On the other hand, we can understand the magistrates' interest in algebra as a new form of dialectic.

In conclusion, we have argued that the French algebrists of the sixteenth century were all but naive in the use of rhetorical skills in the writing of algebra. First of all, rhetoric was an education and an educational ideal for people trained in law. Secondly, rhetoric was at the root of the linguistic debate both in Latin and in the vernacular, where the use of a language for science was of particular importance. Thirdly, rhetoric was, for Peletier and Ramus, a proper means for expressing science. Peletier called the relevant branch "ars poetica", Ramus called it "dialectic", but both interpreted Cicero's rhetoric as the most powerful way to give form to reasoning. Within this framework, Talon and Mignault developed a whole theory of the *quaestio* starting from Cicero's text, thereby giving a more concrete sense of how to apply the modest art of rhetoric to the highest contemplative sciences.

Peletier used *ars poetica* as a universal art, capable of directing the mind in dealing with all mathematical sciences. Algebra was expressed through this art, but was not identified with it. However, the theoretical value of algebra and its universality were a contribution to the later idea of algebra.

Mignault promoted the idea of an art of the *form*, a *universal* art, which enabled one to deal with scientific questions. The fact that this doctrine was taught successfully at a basic level guarantees that it was more than a mere episode. This teaching was an aspect of the context in which algebra began to be conceived as the appropriate replacement for Aristotelian reasoning.

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# **Chapter six**

## **The creation of the history of algebra in the sixteenth century**

### *1. Introduction*

We have seen that the French algebrists of the sixteenth century, in particular Jacques Peletier and Guillaume Gosselin, detached algebra from its Arabic origins and from the Mediterranean tradition of the abacus schools. This separation was determined first of all by the development of a French tradition of algebra, conceived for a milieu of learned culture, and it was also consciously accomplished by the writing of a new history of algebra.

Here I shall discuss mainly the second aspect, i.e. the construction of a history of algebra. But I shall also have something to say about the first, since it too, contributed to the establishment of algebra as an academic discipline. At the center of this twofold process was the study of Diophantus' ***Arithmetic*** from a radically new perspective.

### *2. Humanism and anti-arabism*

Humanism, already in its Italian beginnings, was an attempt to build western knowledge directly on western sources, which means ancient Greece and Rome. This project of refounding was in opposition to the obvious dependence of European knowledge on medieval sciences, the Arabic branches of which being most important.

Consider, for example, a famous illustration of this project of redefinition, the ***De rebus expetendis et fugiendis*** by Giorgio Valla (Venice, 1501). This is the prototype for those encyclopedias which excluded any account of medieval and Arabic learning, and accordingly it gives the advantage to classical sources.[245] Valla, together with his particular erasure of Europe's Arabic past, became one of the main cultural points of reference for the

245 See, for the impact on Italian Renaissance mathematicians, P. L. Rose, ***The Italian Renaissance of Mathematics. Studies on humanists and mathematicians from Petrarch to Galileo***. Genève, Droz, 1975, particularly at the pages 48-50.

French algebraic tradition of the sixteenth century. From this point of view, it is not surprising that humanists developing humanist algebra, such as sixteenth-century French algebrists, tried first of all to disconnect their art from the Arabic sources. To accomplish this, they created classical Greek sources for algebra, and in particular, they created an interpretation of Diophantus.

But this does not tell the whole story. For, both anti-arabism and humanism have a history, and they do not mean the same thing through time. As we shall see, taking this into account allows us to follow more closely the interaction between the content of algebraic works and the mathematician's allegiance to an ancient authority or to a theory of history.

To understand the character of anti-arabism in humanism, we should remember that humanism was born within the tradition of Arabic philology. In fact, we could even say that humanism and western algebra were born together at the court of Frederick the II of Sicily. The recovery and translation of Greek works was introduced in Italy in the same context as the transmission of Arabic mathematics and the writing of Fibonacci. For centuries mathematicians divided into two groups: the mathematical philologists and the practitioners. When, in the course of the centuries, especially after the thirteenth, cultural anti-arabism appeared, and acquired the specific tenor of an emancipation from Arabic heritage, the mathematical philologists adopted it. By contrast, the practitioner counterparts of the philologists (like the Arabs themselves) seldom named their scientific ancestors and did not have an interest in history.

The philologist's interest in history was also an interest in contemporary history. In the fifteenth century, particularly after the fall of Constantinople in 1453, but also earlier,[246]

246 In fact, as Grafton 1981 illustrates, the idea that the fall of Constantinople marked the beginning of humanism as a search for Greek texts was diffused in France by Ramus.

Italian humanism became strictly associated with a group of Greek immigrants who had their own reasons to dissociate themselves from Islamic culture, and were actually in search of a Greek revival from which to advance a possible political rebellion against the Ottoman empire. This is the time and the social context in which Regiomontanus announces the recovery of the Diophantus' manuscript.

By the end of the sixteenth century, it was clear that Europe had regained a cultural position comparable to its position in classical times. Furthermore, the political changes in the Mediterranean had modified more and more definitively the perception of the southern and oriental world. In this respect, the battle of Lepanto, which marked a definite limit to Ottoman expansion, was only the completion of a century-long process of detachment. This undertaking was not unconnected with scientific activity, as is suggested by the presence of the scientist Guidobaldo dal Monte in the entourage of Francesco Maria di Urbino, who took part in the Lepanto expedition.[247] This political event was sanctioned by profound cultural changes, about which we know more as far as the southern countries are concerned. In this sense, it is important to see anti-arabism as a contributing factor to, but at the same time distinct from, anti-medievalism, and at the same time to see it as the counterpart of the extensive assimilation of Arabic science at the universities. Avicenna was in fact at the apogee of his influence in Italian universities in the sixteenth century.[248]

On the other hand, humanism and humanistic mathematical philology also changed significantly in the course of the centuries. Our focus here will be on the transformation of

247 "On the way, however, Guidobaldo felt sick and was detained in Messina." P. L. Rose, ***The Italian Renaissance of Mathematics***. Genève, Droz, 1975, p. 223.

248 Nancy G. Siraisi, ***Taddeo Alderotti and his pupils***, Princeton, Princeton U.P., 1981, especially in chapter four, as well as her recent ***Medieval and Early Renaissance Medicine***, Chicago, University of Chicago Press, 1990.

the notion of ancient origins of algebra, a transformation due to the interaction of the genealogy of algebra with the contemporary theory of history. There existed a specific classical genre reconstructing the illustrious origins of inventions, examples of which were characteristically called *De inventoribus* or *De origine artium*.[249]

In the sixteenth century we find several versions of the various sorts of origins, insofar as the mathematicians used this genre as a repertory of types of genealogy. These transformations are not irrelevant to mathematics, because to each sort of genealogy corresponded to a specific style of work or to a specific theoretical choice. As the algebrists reached for academic dignity, they strove to raise the status of algebra to a discipline and to transform it from the merely practical into the contemplative. The result of their endeavour is that research on notation and on the structuring of a theory of equations typical of French algebrists which gave rise to symbolic algebra.

At first this use of the genre *de inventoribus* implied reference to an ancient authority. Later, as the new mathematics established itself, the ancient author became more an object of historical criticism than a guide to a no longer extant but truer science.

Thus there appear in the course of the sixteenth century two mutually reinforcing processes. One the one hand, history of the disciplines (algebra, but also other arts, e.g. medical practices[250]) was constructed through the genre *de inventoribus* and through increasing historical scholarship focussing on the *national* past, while on the other, algebra

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249 See Brian Copenhaver. "The Historiography of Discovery in the Renaissance: the Sources and Composition of Polydore Vergil's *De Inventoribus rerum*" ***Journal of the Warburg and Courtauld Institutes***, 41 (1978).

250 For the example of Arabic medical learning, see Brian Copenhaver, ***Symphorien Champier and the Reception of the Occult Tradition in Renaissance France***, The Haag, Mouton Publishers, 1978.

was transformed into a new discipline, from a practical, "occult" and secondary art to a discipline of high theoretical status within a specific national context. The development of one increased the credibility of the other. This elevation in status was not only rhetorical but institutional as well.[251]

We may now turn to the construction of the origins of algebra in the texts of mathematicians, starting with Regiomontanus' lecture.

### *3. Regiomontanus' lecture in Padua*

Johann Müller (Regiomontanus), as part of a public lecture presented in Padua in 1463, announced the existence of the manuscript of Diophantus' ***Arithmetic***, while at the same time stating that it contained algebra. It was thus that algebra came to have a Greek origin. About this lecture, I shall only mention that after an expanded version of the usual account of the origin of Greek mathematics (Herodotus, Aristotle), Regiomontanus gives a thorough description of the process of transmission and translation of Euclid, and then of Apollonius.[252] His text is about *de utilitate et de origine artium* and is a typical example of a humanistic piece of the *genre*, following in particular the standard Aristotelian principle that the theoretical part of mathematics is ascribed to the Greeks, whereas astronomy and practical mathematics are attributed to Oriental sources. So, Regiomontanus attributes algebra *also* to the Arabs. This is particularly striking in an author who has extensively used

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251 For the idea combining illustrious origins and progress in another science, see Chiara Crisciani ***"History, Novelty, and Progress in Scholastic Medicine"***, ***Osiris***, 2nd series, 1990, 6: 118-139.

252 "There are also the thirteenth books of Diophantus," Regiomontanus writes, "very difficult, never translated into Latin, in which the whole flower of arithmetic is hidden, i.e. the *ars rei et census*, which today is called Algebra by an Arabic name."

and even "adopted" Arabic trigonometry. But the latter remained a practical mathematics, while algebra was already beginning to be shifted from the practical to the theoretical. To promote this process was, for Regiomontanus, a priority. Furthermore, the philological genre of the lecture and the social identity of the Greek humanists dealing with the Diophantus manuscript could at least provide a reason for his choice of this author as a source. Finally, we should remember that Regiomontanus must have been struck by the richness of Hellenistic sources for mathematics, being among the first witnesses of their discovery.

Regiomontanus did not give a lengthy description of the manuscript, but it was enough to raise the question of the algebraic content of Diophantus' text, which would be published (in translation) only in 1575.

Among the authors who mentioned Regiomontanus' initial attribution of algebra to Diophantus, we should first recall Johann Scheubel, in his successful ***Algebrae compendiosa facilisque descriptio***, published in Tübingen in 1550. The scientific publisher Guillaume Cavellat had this text reprinted in Paris in 1551, and thus bestowed on Scheubel the role, in the French milieu, of propagator of Regiomontanus' thesis concerning Diophantus.

### *4. The first printed algebraic treatises (1494-1556)*

Before turning to our main theme, the sixteenth-century French algebrists, let us consider briefly the group of authors and printed books which precede them. These authors, who include Luca Pacioli, Etienne de la Roche, Girolamo Cardano, and Nicolò Tartaglia, are mostly Italians, and belong to the period in which some freedom of movement between the abacus schools and the universities still obtains. They were therefore practitioners as well as humanists. Accordingly, they pay attention to the Arabic tradition as well as to an integration of algebra with Euclid. They never mention Diophantus.

Luca Pacioli' beliefs about the history of algebra are suggested by his comments on the origins of the word "algebra" itself. In his ***Summa de arithmetica, proportioni et proportionalita***, published in Venice in 1494, he writes:[253]

> Having gotten, with God's help, to the very desired place, i.e. the mother of all cases called by the people "the rule of the thing" or the "Greater Art," i.e. speculative practice: otherwise called Algebra and Almucabala in the Arab language or Chaldaean according to some, which in our [language] amounts to saying "restaurationis et oppositionis." Algebra id est Restauratio. Almucabala id est Oppositio vel contemptio et Solutio, because by this path one solves infinite questions. And it shows the ones which cannot yet be solved.

It is interesting to note that Pacioli considers the possibility that the name of this art could be of Chaldaean origin. Pacioli is therefore the first to draw at least a short history of the topic, and does it according to the *genre* of *de origine artis*.

The contrasting case is provided by *Etienne de La Roche*, who is still working almost entirely within the abacus school tradition. When he writes the first French manual including algebra in 1525, he does not deal directly with the history of algebra, but stays closer to the style of the masters of abacus, citing the numerous contemporary authors which he has "colligé et amassé" but without giving special importance to genealogy.

It is Cardano's text, however, which will serve as the main illustration, since Cardano

253 Gionti con l'aiuto de dio al luogo molto desiderato, cioe la madre de tutti licasi detta dal vulgo la regola de la cosa over Arte magiore: cioè pratica speculativa: altramenti chiamata Algebra e Almucabala in lingua àrabica over caldea secondo alcuni che in la nostra sona quanto che adire restaurationis et oppositionis. Algebra idest Restauratio. Almucabala idest Oppositio vel contemptio et Solutio. perche per ditta via si solvano infinite questioni. E quelle che non fossero solubili ancora le si dimostra.(f. 144)

represented the master link between the practitioners' and the humanist tradition. Moreover, Cardano was also taken as a crucial source by the French algebrists.

The ***Ars Magna*** (Nürnberg, 1545) is innovative in genre, insofar as it is entirely devoted to algebra. It is the first algebra text published in Latin. These innovations notwithstanding, Cardano's idea of the history of algebra is very much in continuity with the content of medieval learning. He writes:

> This art originated with Mahomet the son of Moses the Arab. Leonardo of Pisa is a trustworthy source for this statement. There remain, moreover, four propositions of his with their demonstrations, which we will ascribe to him in their proper places. After a long time, three derivative propositions were added to these. They are of uncertain authorship, though they were placed with the principal ones by Luca Pacioli.[254]

Mohammed-Ibn-Musa, or Al Kwarismi, is therefore specifically identified as the inventor of this art. Cardano reaffirms this elsewhere[255] when he says that in a large paper volume, in fact the ***Liber abaci*** by Fibonacci, "the name of the author of the book which is called Algebra" is Mahomet.[256] This indicates that Fibonacci's attribution was taken for granted by Cardano, in particular for the section on Algebra.

*Nicolò Tartaglia* belongs to this same tendency. In his ***General trattato di numeri e misure*** (1556-60) he mentions Al Kwarismi prominently at the head of a chapter as the inventor of algebra. Tartaglia is in fact the last major figure who does not demonstrate an

254 Shortly after there is the attribution of other solution formulas, including the one to Scipione dal Ferro and Tartaglia.

255 In the 39th problem of his ***Ars Magna Arithmeticae (Opera Omnia***, IV, 374).

256 In fact, also in the printed version of Leonardo Pisano's ***Liber Abaci***, appearing in volume I of Baldassarre Boncompagni's edition of his ***Scritti*** (Roma, 1857) there is a notation *Maumeht* at the beginning of Part III, entitled *De solutione quarundam quaestionum secundum modum algebra at almuchabale* (p. 406). I take this indication from Girolamo Cardano ***The Great Art***. Transl. and ed. by T. Richard Witmer. Cambridge MA, M.I.T. Press, 1968, p. 7.

awareness of Diophantus and who does not feel the need to distance himself from the Arabs.

At the same time, the German tradition was also developing. Michael Stifel published, in 1543, his ***Arithmetica integra***, addressed to a university audience. Again, there is no full-blown history of algebra, but he mentions Geber as the inventor of Algebra: it is defined as "cossa seu Ars Gebri." I want to make clear here that this "Geber", the historical Arab individual of the eleventh century, is the same one who will be appropriated by Regiomontanus.

### *5. The French national style in algebra: Jacques Peletier du Mans*

In contrast to what happened in Italy, mathematicians at the end of sixteenth century in France belonged for the most part to the milieu of jurists connected to the court. We have seen that the Parisian colleges (especially the Collège Royal), the Parliament, the Academies and the publishers were their institutions. We have also seen that there are two aspects of their national style in algebra: "heuristic" rhetoric and the creation of a history. Here we shall discuss the history of algebra.

It is known that historical scholarship developed in France through the impulse of classical studies. Humanistic history had originated in Italy, but its French heirs were aware of the latest, more critical aspects of historiography. Thus, the movement to write the history of France was not only dominated by classical models, but was aware of the need to reconstruct the specificity of this nation, according to the more refined notion of *imitatio*. Among the first authors of the French movement for a national history was Etienne Pasquier, a figure in many ways comparable to Jacques Peletier for his commitment to the founding of a French culture, combined with a profound sense of the classics and an understanding of the limits of French culture itself. In the following years, the historical movement was even more clearly connected to juridical studies. François Hotman, (who had emigrated from France but continued to be very productive), François Baudoin, and Pierre Ayrault are the most typical representative of this second phase. This period, like the one before it, was, characterized by discussions on language and the history of language as a crucial dimension in differentiating ideas and methods in history.

The debate opposing Roman law to customary law involved a discussion of not only the difference of circumstance but also different modes of textual representation. This awareness of course had its origins in Italian humanism. In the same way, the national orientation of legal thought confirmed the preexisting movement for the writing of a national history. Furthermore, the interaction between historical and legal thought represents not only a philosophical transition, but is also reflected in the interest of a group. In the second half of the sixteenth century the great majority of books owned by jurists were history books, and among these, mostly books of national history. Since these jurists were particularly important for the patronage and promotion of mathematics in general and algebra in particular (Cujas is mentioned among the patrons of a Diophantus' edition), we can associate this historical culture with the specific genre of the history of algebra.

However, we should remember that this picture corresponds to Paris at the end of the century, at the time of Gosselin and Viète. Only at that point do the advances in juridical thought determine changes in the notion of history and of historiography. It is worth remembering that the end of the century is the time in which the juridical milieu is very powerful. But that is already the second phase of French algebra.

Peletier devoted much of his attention to the question of the history of algebra. This is quite clear from the introduction to his algebra text, ***L'Algèbre*** of 1554, where he goes far beyond the lists, etymologies, and off-hand references of the authors we have seen before.

First of all, he refers to the Arabic sources which have been traditionally acknowledged. To these he adds Pacioli and other writers of the early sixteenth century:

> ....le premier inventeur de cet art, selons aucuns, fut Geber Arabe: et se fondent sur la raison du mot, composé d'un nom propre et d'un article Arabiq, eui est Al: lequel se prepose communement aus motz de la langue: comme Alcabice, Albubater, Alcandan, Alquemie; et assez d'autres que nous avons d'eus, principalemant an Astrologie. Selon les autres, fut un Mahommet fiz de Moise Arabe: Lequel, comme dit Gerome Cardan, Millanoes, après un Leonard de Pesare,[257] an a lessé quatre chapitre ou regles avec leurs Demonstracions: lequelles ne se trouvent publiquement, que je sache. Frere Lucas Pacciole Florantin, l'a mise an son vulguere, Après lui, Cardan l'a ecritte en Latin: E puis Michel Stifel Allemant.[258]

Later, Peletier mentions Diophantus, using Scheubel as his source:

> I'è encores vu le liure de Ian Scheubel, Matematicien de Tubingue: lequel attribue l'invancion de cet art à un Diophante Grec, qui an a lessè treze Liures, au rapport de Ian Demonroe,[259] fameus Matematicien de notre tans, dines certes, de grande conquisicion, s'iz etoèt d'avanture recouurables.

But Peletier has his own opinion of the subject of the origins:

> An telle diversité d'opinions, me souvient d'an dire la mienne incidammant: C'est que je ne pense point que cet Art, ni la pluspart des autres, doivent leur invention à un seul auteur. Mais bien que quelqu'un en fait l'ouverture fort

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257 Obviously, Peletier is thinking here of Leonardo Fibonacci of Pisa.

258 Peletier continues: "lequel allegue an son liure un Crestofle Ianver e un Adam Ris, tous deux Allemans, qui l'on redigee an leur langue. I'è ancores ouì dire de Pierre None, Matematicien de Lisbonne an Portugal, qu'il avoèt aussi trettee en son langage Espagnol: Mes je n'è vu son liure, nomplus que des deus Allemans: e croè qu'il n'è ancores publié. Aquez certes èt due grand louange."

259 Peletier is thinking of Regiomontanus.

> rude et malpolie, peut-être sans penser qu'il s'en dut ou put faire un Art: et puis de main en main, et par longue circuicion, de tant et continuelles exercitations de l'esprit, les hommes ont donné forme, reigle et ordre à ce qui n'avait rien de tel. Et enfin les Arts se sont trouvé rédigés et unis, mais par tant d'intermissions, (car la longue durée a besoin de long ouvrage et de long achevement), que nul des mortels n'en peut avoir seul la préminence. (*L'Algèbre*, p.3)

Peletier had already expressed a similar point of view on the accumulation of mathematical knowledge with respect to the history of arithmetic. In his ***L'Arithmétique*** (1551) he had posed a question of particular importance with respect to the Greek heritage much like the one Descartes raised later in the ***Regulae***: "comment il se peut faire que les anciens ne nous aient laissé par écrit la prattique et usage de l'Arithmétique?" But where Descartes will claim that the ancients had had analysis and algebra and they had hidden it, for Peletier there was another explanation: "les inventeurs ne tendent pas à écrire suivant l'"ordre métodique", but rather following the order of invention itself. Writing, therefore, came only later, when ease in the art and need in the practice made it possible:

> Mais a la fin, croissans toujours les affaires et traffiques des nations les unes avec les autres, la commodité et nécessité, qui ouvrent les esprits des hommes, leur a enseigné à établir un stile, qu'ils ont disposé par état, peu à peu, quand chacun a apporté sa part d'invention au bureau, pour soulager ceux qui n'avoint loisir de vaquer à la Théorique." (Proème au 4e livre de l'*Arithmétique*)

In fact, Peletier found himself at the juncture of two traditions, and he was actually aware of undertaking to bridge between them. Writing in French, he introduced the arithmetic and algebra coming from the abacus schools in Italy and Germany to the cultivated milieu of the court and the university. We have seen that in his ***Dialogue de l'Ortografe*** he had written that this was a way of following the example of the Arabs[260]

Nos mathématiques ne furent jamais mieux au net, qu'elles sont de présent,

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260 See quotation from the ***Dialogue de l'Ortografe*** in chapter 1.

> ni en plus belle disposition d'être entendues en leur perfection. Et par ce que leur vérité est manifeste, infallible et constante, pensez quelle immortalité elles pourraient porter à une langue, y étant rédigées en bonne et vraie méthode. Regardons même les Arabes, lesquels encore qu'ils soient reculés de nous et presque comme en un autre monde: toutefois ils s'en sont trouvés en notre Europe qui ont voulu apprendre le langage, en principale considération pour l'astrologie, et autres choses secrètes qu'ils ont traité en leur vulgaire, combien qu'assez malheureusement. Car on sait quelle sophisterie ils ont mêlée parmi la médecine et les mathématiques mêmes. Et toutefois ils ont rendu leur langue requise en contemplation de cela. Avisons donc à quoi il peut tenir que nous n'en fassions non pas autant, mais sans comparaison plus de la notre? (Jacques Peletier du Mans, *Dialogue de l'orthographe*,1550, pp. 117-118)

Thus, in writing his own treatise on algebra, Peletier was able to acknowledge that the Arabs, at least as a people, had contributed to the invention of algebra. In this attribution he followed Herodotus' criterion for the invention of the arts, according to which they should be ascribed to a people, and not to an individual, to avoid mythical figures. But besides this philological choice, Peletier shows, here as elsewhere, his awareness of the new French history of law. Etienne Pasquier would publish his ***Recherches*** only in 1560, but both authors clearly express similar points of view about history and language. This is what has been later defined as the beginning of cultural relativism, based on an awareness of the specificity of Roman culture and then of French national identity.[261] While the beginnings of this view belong to Italian humanists, they had been introduced in France by Guillaume Budé, and this position will be expanded in the debates about the conflicting traditions in law, the transition being represented by François Hotman, who worked on language, history and law.[262] The supporters of customary law over Roman law, expressed the priority of

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261 An excellent survey is given in George Huppert ***The Idea of Perfect History. Historical Erudition and Historical Philosophy in Renaissance France*** Urbana, University of Illinois Press, 1970.

262 See in particular Donald R. Kelley, ***François Hotman: a Revolutionary's Ordeal*** Princeton, Princeton U.P. 1973.

customary law over the authority of the ancient texts. In this view laws appeared ."..not as a manifestation of reason, but as the accumulation of many human judgments", as Pierre Ayrault wrote in his ***De origine et auctoritate rerum iudicatarum*** (Paris 1573). Ayrault's ***res iudicatae*** can be compared to the concept introduced by Peletier of "accumulation of knowledge" in the discipline of algebra, without the need for a *universal* ancient authority.[263]

***6. Jean Borrel, (hellenized as Buteo)***

Jean Borrel, or Buteo, published his ***Logistica*** in Lyon in 1559. Borrel's point of view about the history of algebra is made explicit already by the title of his work, for he used a Greek name. The word "Logistic" had been used by Plato both as the term for calculation and for the theory of calculation (the four operations). By contrast, the Neoplatonists had used it in opposition to arithmetic to indicate the arts of calculating as distinguished from the science of numbers. Buteo uses the term in yet another way. For him, "logistic" refers to practical arithmetic, i.e. the study of the four operations, a meaning that had become common thanks to the diffusion of treatises in the abacus tradition, where discussions of practical arithmetic included calculation with numbers of all kinds (e.g. fractions) and a special chapter for cossic numbers (which we would call algebraic numbers). Also at this level, Buteo introduces a new terminology, calling the cossic numbers *quantitates (geometricae)*. Even the name of the discipline itself, algebra, is changed into *quadratura*. He starts the section on algebra with the following statement:

263 While Ayrault's statement actually came twenty years later than Peletier's ***L'Algèbre***, this idea was expressed in a more extended form by earlier authors, such as Etienne Pasquier. The complex process of evolution of these theories is explained in Donald R. Kelley ***Foundations of Modern Historical Scholarship. Language, Law and History in the French Renaissance*** New York, Columbia U.P. 1970.

> There remains to be added to the top, as a crown, that type of reasoning which is called popularly by the Àrabic name of Algebra (qui vulgo et Àrabica voce dicitur Algebra). I prefer to call it *quadratura*. In fact this is a rare and subtle practice which the Logista takes from the Geometer as a help. (p. 117).[264]

Borrel makes mention of the Arabic origins of the word algebra only to displace it with a distinctively Latin name. Later, his criticism of the Arabs becomes less subtle and he advances a view soon to become a European commonplace: the Arabs are deficient in scientific work, their presentation is obscure and their language impure. Against this general background he writes:

> ...the utility and the intelligence of quadratura is accompanied by a specific difficulty, which derives more from the defect of the propagators than from the nature of the thing. For those, really ignoring the method of the disciplines, going far in the roughness of words and things, involve and trouble everything to the point that nothing could be more confused, and accumulating the clouds they obscure the senses of the readers.(pp. 117-118)[265]

The "ignorant propagators" here are not only the Arabs, but Pacioli and La Roche, who stand accused of being like Arabs. Thus, by means of a general anti-arabism Buteo impeaches the whole abacus tradition through its major late representatives. At the same time, the link to the ancients is affirmed by Borrel's faith that Euclid has in fact transmitted

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264 His expeditis quae sunt ex usu numerationum communi, restat ut eum ratiocinandi modum operi summo veluti coronidem adiiciam, qui vulgo, et Arabica voce dicitur Algebra. Ego autem, prout revera est, quadraturam dicere malo. Opus sane rarum, et exquisitum, quod a Geometra Logistices, subsidio quodam mutuatur.

265 Sed utilitatem, et intelligentiam quadraturae dissicultas praecipua comitatur, magis quidem tradentium vitio, quam rei natura proveniens. Hi nanque disciplinarum methodon prorsus ignorantes, verborum, atque rerum late vaganti barbarie, sic implicant, atque perturbant omnia, ut nihil possit esse confusius, unde legentium sensus, conglobatis veluti nebulis, obumbrant.

the art in his tenth book. According to him, this transmission had gone unnoticed because it can only be understood by the reader of the ***Elements*** who has become proficient in the previous books. In conclusion, Borrel's contribution to our theme is twofold: he hellenizes algebraic terminology, and he considers algebra to be contained, at least implicitly, in Euclid. Furthermore, his anti-arabism is the most explicit: the Arabs are deficient in scientific work, their presentation is obscure and their language is impure, and the same is true for their abacist successors. In this, Borrel's attitude is similar to that of philologists of the previous centuries. Borrel, like all humanists working on Euclid, had spent a significant portion of his life trying to discern, in the texts, what belonged to Euclid and what was a later addition. In this sense, any transmitter is responsible for corrupting the text.

### *7. Ramus*

Ramus is, I think, particularly interesting on this matter, and not only because of his importance for mathematical education. His views on the genealogy of algebra are multifaceted and complex, and seem in some respects to be contradictory. Here I want to stress that when all of his published works are taken into account, including the various editions of the ***Algebra***, two rather different views emerge.

The first view seems generally to fit the pattern established by many of his contemporaries, in which Arab authors are displaced in favor of Greeks. The second view revisits an older mythical account of ancient knowledge that is more inclusive, to the point even of including the Gauls among the ancients. On careful inspection we shall see that these two views are not necessarily incompatible.

Ramus deals with the history of science in many of his texts, but he addresses the history of algebra only in one work published during his life. In his ***Scholarum mathematicarum libri unus et triginta*** (1569), the first book is devoted to a vast history of the mathematical sciences. Ramus clearly aims to be comprehensive, and it is significant that he includes no reference to Arabic or Oriental authors, building entirely on Greek sources. But amongst these, he mentions a new one. In a list of (the great) Alexandrian mathematicians, he takes care to inform the reader that there exist the six books of Diophantus in Greek.[266] This places Ramus squarely within a contemporary tendency which gives great weight to Diophantus.

At the same time, the elimination of all but Greek sources is also a change with respect to the genre of *de origine artium*. From the Greeks came the theoretical sciences, but in the ***Scholae*** even the practical sciences are not ascribed to Oriental authors as they are, for instance, by Regiomontanus and, before him, in Aristotle. Ramus can attribute algebra to the Greeks alone because the distinction between theory and practice is no longer important for him in its classical form. This distinction is rearranged so that each science overlaps with its corresponding art, and thus the *use* of each, which can be *ad contemplandum* or *ad agendum*, becomes the crucial distinction. Furthermore, Ramus remains confident in relying on the Greeks alone because he does not treat them as absolute authorities, but rather as interlocutors to be engaged polemically. This is clear, for example, in the way he criticizes Proclus.

Ramus' second version of the genealogy of algebra becomes apparent when we examine the various editions of a treatise, ***Algebra***, known to have been written by Ramus

266 Diophantus cuius sex libros, cum tamen autor ipse tredecim polliceatur, graecos habemus de arithmeticis admirandae subtilitatis artem complexis quae vulgo Algebra Àrabico nomine appellatur: cum tamen ex authore hoc antiquo (citatur enim a Theone) antiquitas artis appareat.

but published anonymously. The first date of publication was 1560 (henceforth, ***Algebra**$_1$*).

The book was then reprinted three times posthumously. These three editions (henceforth, taken together, ***Algebra**$_2$*) were brought out in 1586, 1592 and 1599 in Frankfurt, by the publisher of the original edition, André Wechel,[267] and them included corrections by Lazar Schoner.

What interests us here, however, is that while ***Algebra**$_1$* contains no history of algebra, an elaborate genealogy is included in ***Algebra**$_2$*.

Considering this second genealogy contained in ***Algebra**$_2$*, in contrast to the one in the ***Scholae***), we must proceed with some caution, for we do not yet know with certainty that Ramus is its author. And if he is not, we have still to ascertain the extent to which Schoner the editor cared to follow closely Ramus' ideas on this matter. But, despite these qualifications, it is nonetheless significant that we find the following view attributed to Ramus in 1585, which appears to be in partial contrast with the history proposed in the ***Scholae mathematicae***. So, let us now trace this second view.

***Algebra2*** ascribes the invention of the art of algebra to a wise man described as geber:[268]

267 In fact, the second, third and fourth editions were published by the heirs of André Wechel, C. de Marne and J. Aubry, in Frankfurt. The Bibliothèque Nationale has only the 1592 edition, but I was also able to consult the Wolfenbüttel library for the 1586 edition and the New York Public Library for the 1599 edition.

268 De his numeris dictum est 5 cap. de figuratis, ubi eorundem etiam fuit numeratio quaedam, cui frequens adhibita est resolutio, sumpto quocunque valore lateris. (...) Nomen Algebra Syriacum putatur, significans artem et doctrinam hominis excellentis. Nam Geber Syris significat virum, idque nomen interdum est honoris, ut apud nos Magister aut Doctor. Etenim insignis mathematicus quidam fuisse fertur, qui suam algebra Syriaca lingua perscriptam ad Alexandrum magnum miserit, eamque nominaverit Almucabalam, hoc est, librum de rebus occultis, cuius doctrinam Algebram alii dicere maluerunt. Is liber hodieque magno precio est apud illas eruditas Orientis nationes, et ab Indis harum artium perstudiosis dicitur Aliabra, item Alboret, tametsi proprium autoris nomen ignoretur. Algebra vero a Latinis quibusdam dicta fuit ars rei et census, ut est apud

> The name of algebra is thought to be Syriac, signifying the "art and doctrine of an excellent man." Now *geber* in Syriac signifies "man"; it is often a title of honor, as "master" or "doctor" with us. For, there is said to have been some unknown mathematician who sent his algebra, written in the Syriac tongue, to Alexander the Great, and he named it *almucabala*, that is the "Book of Occult things." Others preferred to call his doctrine algebra. This book is still today very precious among the erudite nations of the East, and it is called by the Indians, who are very studied in these arts, *aliabra*, or *alboret*, since they are ignorant of the origin of the proper name. Algebra has been called by some Latin *Ars rei et census*, as in Regiomontanus. By the Italians it is called *ars de la cosa*, by others *cossa*. Many schools today neglect to note how many names, or perhaps even more, algebra has had, in what high regard learned men of all nations have held it and what the loss of the doctrine would mean.

Note that the central character of the first genealogy, Diophantus, is entirely absent here. Instead, the story of the "geber," who is not even mentioned in the ***Scholarum mathematicarum***, is told in etymological, if not historical, detail.[269]

Our surprise at the absence of Diophantus is amplified by another clue the author provides. The passage where he cites the Latin name for algebra is drawn from the same lecture in which Regiomontanus had announced, a century earlier, the existence of the manuscript of Diophantus. Furthermore, whether the author is Ramus or Schoner, he must have known at least Scheubel's ***Compendiosa descriptio*** and Peletier's ***L'Algèbre*** and his list of inventors. Both of these important contemporary algebra manuals gave a special place in the history of algebra to Diophantus.

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Regiomontanum. Ab Italis ars de la cosa, ab aliis cossa. Quibus tot nominibus ac fortasse pluribus etiam palam fit, quanti fuerit haec doctrina apud doctos omnium gentium homines quantaque cum iactura doctrinae plerisque in scholis hodie negligunt.

269 The most common copies of this text are of the edition by Lazar Schoner from 1586 and 1599. However, there are no significant differences between the Schoner edition and the anonymous edition printed by André Wechel in Paris in 1560 under the title ***Algebra***.

In spite of the apparent contradiction with the earlier version, in order to explain the absence of Diophantus from the genealogy of ***Algebra₂***, it will help to assume that Ramus is the author. And there are good reasons to believe that this was indeed the case.

At first blush, one might ascribe the attribution given in ***Algebra2*** to the choice of *genre*. The ***Algebra*** (in its various editions) is not an innovative manual by comparison with contemporary French algebra manuals. Its author remains within the tradition of the German Coss. Thus, while we must admit that ***Algebra2*** in particular introduces as a new element the treatment of *de origine*, the author nonetheless follows the essential contours of a line laid down by Stifel.

In following this tradition, the author of ***Algebra₂*** displays creative flair of a particularly telling sort. The "geber" here is no longer the historical and Arab "Geber" (of the eleventh century) mentioned by Stifel, the author of the trigonometrical works appropriated by Regiomontanus. In this genealogy the "geber" is interpreted as a *magus*, belonging to the Semitic melting pot which is identified with the *prisca theologia* or Chadean wisdom.[270] While the Syriac name could indicate the third century A.D., the reference to Alexander transforms him into a mythical figure of the alchemic type. Chaldaean wisdom was a *topos* in the philological genre *de inventoribus*. In particular, Jewish philologists and thinkers such as Philo of Alexandria, as well as some Christians such as Clement, had held Abraham and the patriarchs in general as the inventors of all knowledge, which later passed to Egypt and then to Greece.[271]

270 See in particular C. Schmitt "*Prisca theologia* e *philosophia perennis*: due temi del Rinascimento italiano e la loro fortuna", dans ***Il pensiero italiano del Rinascimento e il tempo nostro***, ed. G. Tarugi, Firenze, Sansoni, 211-236, as well as "Perennial philosophy: from Agostino Steuco to Leibniz", ***Journal of the History of Ideas 27****,* 505-532.

271 See the article, already cited, by B. Copenhaver.

Now we come to the main point. The way that Ramus willfully "misreads" Stifel and the German Cossic tradition to point in ***Algebra*₂** towards the mythic "geber" is entirely in keeping with his philosophical standpoint. Simply, Ramus gave great importance to the Oriental Chaldaean wisdom, which he understood as the source of Greek knowledge. Already in his ***Gramere*** in 1562 he had maintained that the Gauls did not need to *imitate* the ancients, because, from the point of view of the language, the Celtic tradition (and particularly the Gauls) was the source of the Chaldaean.[272] In this light, Diophantus need not be mentioned; he was a transmitter of the algebra rather than an author *per se.*

This way of reading Ramus' complex view of the genealogy may surprise us for another reason. It is quite likely that he was acquainted with the actual content of Diophantus' text, the first manuscript of which came to France, and into the hands of his associate Gosselin, around 1570. That is why, we may assume, when Ramus writes his history of mathematical disciplines in the ***Scholae*** he is able to indicate Diophantus as the author of algebra. But, does this not suggest a change in the *opposite* direction? That is, should not the attribution shift *from* the "geber" *to* Diophantus, representing the predictable passage from a mythical source to an historical character who lived in Alexandria? This would have allowed Ramus to move from *magus* to mathematician, from occult art to calculation.

In fact, what I want to stress is that the order of attribution actually present in the texts, i.e. from Diophantus to "geber," and the reading of it I have proposed, are not

272 See, on this sixteenth-century *topos* and its rôle in Ramus' philosophy, the proceedings of the meeting "Pierre de La Ramée" in ***Revue des sciences philosophiques et théologiques*** 1986, 1, and especially the texts by P. Magnard "***L'enjeu philosophique d'une grammaire***" and K. Meerhof "***Ramus et Cicéron***", as well as by C. Vasoli "***De Pierre de La Ramée à François Patrizi. Thèmes et raisons de la polémique autour d'Aristote***.

incompatible, once the whole picture is taken into consideration. For, again, we should remember that Greek mathematics itself was conceived as dependent upon the old Mesopotamian wisdom, and this was the view particularly of Ramus. So, while we cannot be sure whether he actually changed his mind, we know that from his point of view the two stories are not in contradiction, but actually can acquire a sense which is closer to sixteenth-century cultural tendencies. In fact, the genealogy provided in ***Algebra2*** could indicate a stronger committment to the construction of a French national past.

The genealogy given in ***Algebra2*** should be compared with the work of a contemporary author writing in verse about mathematics, Guy Lefèvre de la Boderie. He was an esoterist and an Orientalist but, like Peletier and many others, wrote philosophical (i.e. scientific) poetry. He published his ***Galliade ou la Revolution des arts et sciences***, in Paris in 1578, though from the point view of the myth of foundation, he collected material that had been common in the Parisian academies for years. That is, simply, a new version of the Gallic myth of Lemaire de Belges.

In the ***Galliade***, La Boderie deals extensively with mathematics. He devotes a whole section to Archimedes, and then concludes:

> Donques nos vieux Gaulois, non les Egyptiens,
> De la Mathématique, et des Arts Anciens
> sont premiers inventeurs: et la source gardée
> Es hauts mont d'Arménie, et puis en Chaldée.

La Boderie goes on to explain how knowledge, thanks to Gomer, the Gallic Hercules, went from Gaul to Chaldea and then to Egypt on one hand, and to Italy and Gaul on the other. This last passage was, in La Boderie's account, in fact a return to the origins. This implies, of course, that Greek knowledge was derivative rather than original.

We can see that here myth exists in its proper sixteenth-century medium, neoplatonic scientific poetry. It is fitting that in this context the authors of contemporary French mathematics are mentioned, from Peletier and Forcadel to Ramus. But there is an even more specific point concerning the mythical significance of algebra. According to La Boderie's poem, the Phoenician letters had their origin in Gaul. In recent times, he writes, there is still a new role for

> L'usage et les secrets de la mistique lettre
> Des vieux Pheniciens, et lettres et secrets
> Qu'eurent les Grecs de nous, et non pas nous des Grecs.
> Ainsi au fil des ans ceste Ecriture ornée
> Qui en Gaule nasquit, en Gaule est retournée.

We can see in La Boderie the ultimate representation of the connection between France and classical antiquity. Gaul is the source of Chaldaean wisdom, so that what is the oldest is also the closest to home.

There are at least two consequences of this approach. First, the Greek authors are not the inventors, but the depositories of ancient wisdom. They can therefore be treated as Ramus treats Aristotle, which is to say, at times as an impostor. Secondly, the French, being heirs of the Gauls, were in the best position to correct the Greeks. They could do what had been impossible for medieval authors, including the Arabs.

In this same period in which La Boderie writes the ***Galliade***, however, historical scholarship, and in particular critical philology applied to mathematical texts, was putting in place a new way to evaluate ancient tradition. The idea of a Golden Age, whether represented by the Greeks, or by the Chaldaeans, or by the Gauls, was set aside. It might still be admitted that the Greeks created the sciences, but the moderns were, from this developing point of view, taken to be more advanced than even the Greeks. Such a position was expressed by Joseph Scaliger,[273] taking aim precisely against Ramus. As we are about to see, it was also characteristic of the French algebrists once they set aside the mythical Diophantus and began to work towards the actual recovery of his texts.

### *8. Diophantus recovered: Gosselin and Vìete*

Let us take up again Gosselin's version of Tartaglia, published in 1578. As we know, Gosselin was connected to the court through the Académie de Baïf, which constituted part of his audience. Thus, we can expect his version of the history of algebra to be more similar to Peletier's than to that of Lefèvre de la Boderie more than to Peletier. But we also know that he studied Diophantus, and was more aware of its content than any other Frenchman before him, and thus able to determine whether its content should be considered algebra or not. In the dedicatory letter to Marguerite de France, queen of Navarre, he writes:

> Cette divine Algèbre en laquelle une Royne d'Alexandrie Hypatheie a esté si excellente (ainsi que le dit Suide) qu'elle a osé commenter sur le plus difficile livre, qui pourra jamais estre composé, à savoir sur le Diophante, qui traite de cette partie, neantmoins que ses commentaires ne soyent venus iusques en noz mains.

Hypatia had been recently mentioned by Xylander in his edition of Diophantus published in Basel in 1575. Howevever, Gosselin did try to go further than Xylander in determining what Hypatia had written, for he gives her more importance than Xylander in his own dedicatory letter.[274] To be sure, Gosselin mentions Hypatia as a woman

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273 See Anthony Grafton ***Joseph Scaliger. A Study in the History of Classical Scholarship*** I, Oxford, Clarendon Press 1983, especially chapter VII: "Scaliger's ***Manilius***: from Philology to Cultural History", pp. 180-226.

274 Xylander 1975, Epistola f. 4. Suidas writes: "Egrapsen hypomnema eis Diophanton... tov astronomikon kanona, eis ta konika Apolloniou upomnema." This passage gave rise to diverging interpretations, given that Diophantus did not write on astronomy. The most recent interpretation suggests that Hypatia commented upon Diophantus, Ptolemy, and Apollonius. Suidas goes on to

mathematician, in honor of the queen Marguerite, to whom the book is dedicated. But it is also a way to remember that algebra existed and flourished in Alexandria, a thesis completely extraneous to Tartaglia's treatise, of which this text was supposed to be the French version. Hoever, Gosselin also gives some space to other authors. He cites the Al Kwarizmi *qu'on dit être inventeur de l'algebre*. He also mentions *Diophante, qu'aucuns aultres disent être inventeur de l'algèbre*. Finally, Gosselin criticizes Tartaglia for not having mentioned the *auctores* of the discipline. Tartaglia had in fact mentioned *only* Al Kwarizmi. This seems to be, above all, a way to stress the importance, in the new conception of the algebraic treatise, of a section *de origine*, which should be as rich as possible, especially in ancient sources, without excessively privileging the Diophantian origins. Yet, Gosselin was working on Diophantus. He was connected to the Classical and scientific scholars belonging to the Parliament of Paris. They had commissioned from him an edition of Diophantus, and Davy du Perron had lent him his manuscript of the ***Arithmetica***. At the beginning of the *Ars Magna*, there is a section on the inventor of algebra. Gosselin writes that some attribute the invention to the historical Geber (the eleventh-century trigonometrist), some others to Mahomet son of Moses (Al Kwarizmi), others to Diophantus the Greek. But he declares himself to be convinced that this art existed before these times, because, in his reasoning, it is the science which should acquire the highest dignity, since it allows the algebrist to solve all problems.

Thus he does not take Diophantus as the only source, but he mentions a series of main authors, including the Arabs. It seems that with Gosselin, as it becomes clear later with Viète, Diophantus' text is actually taken into account in such detail that its algebraic

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collect classical information about Hypathia's life.

content is no longer exaggerated. Instead, the text is used to elaborate and extend the discipline further. In addition, Diophantus' role as mythic founder ceases to be necessary as it had been for Regiomontanus, while the myth of origin of the type brought forward by Ramus was no longer appealing. There was more than one reason for this, including the increasing awareness of contemporary technical achievements, as well as of philological discoveries, in particular the actual knowledge of the text of Diophantus. In addition, the new conception of history also plays a part, and (again) Gosselin should be understood within the cultural framework of this contemporary historical scholarship. We could say that what gave the French algebrists the self-legitimation to transform Diophantus from a mythical *auctor* into an object for critical analysis was the theory of the independence of French tradition proposed in law and history of law by François Hotman (***Jurisconsultus, sive de optimo genere juris interpretandi***, 1559), Etienne Pasquier (***Recherches de la France***, 1560), and François Baudoin (***De institutione historiae universae et eius cum jurisprudentia coniunctione***, 1561). As Le Caron put it in 1566, "Frenchmen, you have enough examples in your history without searching those of the Greek and the Romans."[275]

Whether purists like Cujas, who was also Gosselin' patron, or "medievalists", sixteenth-century jurists were learning to apply their understanding of geographical and historical relativity to law and customs. This process led to the famous universal histories by Jean Bodin (***Methodus***, 1566) and Louis Leroy (***De la vicissitude ou variété des choses en l'univers***, 1575), who explicitly intended to interpret the differences in things (we could say "facts") in terms of the differences in places.

When Leroy wrote the ***De la vicissitude***, the present and the recent national past had

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275 See Louis Le Caron ***Réponases et décisions*** Paris, 1566.

been evaluated and found to be rich enough to allow a magnanimous acknowledgement of the Arabic heritage, while still permitting him to give the greater weight to the rôle of France, and of Europe, in the development of the sciences. Arabic heritage had, following Lepanto, by then become less of a danger in fact and more of a new mythical presence.

As the basis for the actual interpretation of Diophantus' text, this revised historical perception was as important as the increased philological sensibility. Both guided readers to focus on the mathematical content, to develop a critical interpretation of it, and finally to elaborate it in new directions.

This form of humanism, with its particular legal character, typified French mathematicians. And it is, I argue, the main cultural reason for the special events which occurred in French algebra from Gosselin and Viète up to Descartes. If we compare Gosselin's (1577) and especially Viète's work (1591) on algebra to the other contemporary elaborations of Diophantus, those by Bombelli (1572) and Stevin (1585), we find important differences worth articulating. In particular, we can see that the French writers work with much greater freedom in separating the algebraic content both from the abacus tradition and from Diophantus' text, in order to introduce it into a totally difference context, that of a *subtilior arithmetica* in the case of Gosselin, or of an *ars analytica* in the case of Viète.

### *9. Conclusions*

The algebrists of sixteenth-century France could have taken seriously the previous Italian tradition, dependent as it was of the abacus schools. Cardano, whom they used directly as a source for algebraic techniques, represented very clearly the Arabic origins of algebra. Accordingly, they could have accepted the Arabic authors as inventors instead of propagators, or they could have taken as a myth of origin the Mediterranean centers of learning, both humanistic and mathematical, for instance, Sicily. This did not happen. The reasons for this can be seen in religious, political and economic history. However, we can see rather clearly the cultural options available to these authors, and the way in which they made their choices. This allows us to connect their scientific and their humanistic work.

Regiomontanus introduced into public knowledge the existence and the content of Diophantus' manuscript, thanks to his connection to Bessarion and to the recovery of manuscripts. But the Italian group of algebrists (and La Roche) were still oriented by their abacus school context. So, for them, humanism meant making a *summa* of the Arabic tradition and connecting it to the Greek corpus.

What happened, instead, in France? Peletier introduced a form of *medievalism*, insofar as the recovery of national past allowed for an acknowledgment of the more recent, medieval tradition. Thanks to the beginnings of historical relativism, recent medieval tradition could be admitted without a loss of national identity. At the same time, mathematics was, in Peletier's mind, like language, the product of a people and not of a person.

Ramus' thesis on the history of algebra, as expressed in the ***Scholae mathematicae***, with its familiar emphasis on Diophantus, fits perfectly with the side of his work that stresses his new idea of *discipline* as that which should arise from a profound revision of what the Greeks had transmitted.

On ***Algebra2*** we find a "Ramist" construction of a *mythical* national past. This can be explained first of all on Ramus's own grounds, since all of his philosophy implied the existence of an Oriental knowledge of divine origin as the basis for Plato and Aristotle.

Secondly, this myth finds a close parallel in La Boderie's reconstruction of the myth of the Gauls.

The time in which the Diophantus manuscript becomes available in Paris, with Gosselin and Viète, is also the time in which Pasquier's notion of history, which had been "anticipated" by Peletier, finds a new formulation in universal histories by Bodin and Leroy. The result is cultural relativism, at least for a small group who shared these interests. But in this perspective, the construction of a national past legitimized the national present, giving value to its contribution to the development of the sciences in Europe. This is the time in which the history of Greece is represented as a part of the history of western culture, which is to say that ancient Greece is the early Europe. This is a time in which the text of Diophantus could be read and discussed in the original. Yet, this access to the text came at just the moment when his authority started to be less crucial in the legitimation of the discipline.

The interaction between national and *de inventoribus* history and the development of a discipline is typical of the process which changes the status of that discipline. This has happened in other times and places, and for other disciplines. For example, Brian Copenhaver has made similar points concerning the development of medicine in sixteenth-century France.

Historians of science have wondered if the revival of the genre *de inventoribus*, or *de origine*, is not the sign of a gap between Renaissance humanism and Renaissance science. I hope I have made clear that the appearance of this humanist genre in algebraic treatises reflects a moment of *interaction* between humanism and science. The importance of origins for humanistic authors led, not to a gap, but to a conjunction of historical and mathematical learning.

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# **Conclusion**

I started with the assumption that a key role in the transformation of algebra between the sixteenth and the seventeenth century was played by the French tradition of algebra, which provided not only the constitution of an academic discipline, but also the elaboration of a theory of sciences in which algebra was determinant.

If we know that this transformation took place in France with Viète and Descartes, we may wonder why it was precisely the French milieu that gave rise to this discipline, even in the absence of a medieval algebraic tradition comparable to the one present in Italy or in the German countries.

I therefore studied the two main French algebrists of the sixteenth century before Viète, i.e. Peletier and Gosselin, in connection with the culture of the more specific milieux for the production and the circulation of algebra in France. These were the Court and the Academies, the Collège Royal and the Parliament of Paris, and they constituted the main source of authors and the main readership for algebra.

The first, preliminary goal of my research was to bring to light the mathematics of these two authors and to understand their motivations as well as the scientific "agendas" they might express. *Scientific* here is used of course in a larger sense, which also applies to the scientific academies of the century. This notion of science is, as we have seen, strictly related to the transformations occurring in rhetoric.

The first result is a greater understanding of the differences between the two algebrists, both in style and in mathematical results. This led me to establish a periodization of French algebra in the sixteenth century, both from the point of view of the theory and from the point of view of the social context. More particularly, in the first phase, Peletier represents the introduction of practical arithmetic and algebra at the court, within a program for the promotion of French as a scientific language. To write a book of algebra in this context lead Peletier to transform the discipline in depth, thereby creating a French tradition characterized by three sorts of innovations: the development of symbolism, the importance given to the notion of equation and to the classification of equations, and a first generalization of the notion of problem.

The second phase of the French algebraic tradition is characterized by the use of Latin, hence by a new change in audience, moving from the Court to instruction of the Collège royal and to the new movement in the academies. At the same time, the return to Latin is a further reference to the juridical culture. The model for this milieu was the "high" rhetoric of the Parliament of Paris, whose eloquence was not limited to a strictly juridical debate. More specifically, rhetoric was understood in an extended, Ciceronian sense, and included the whole range of argumentation, and all degrees of rigorous thinking. Algebra was then integrated in this extended juridical rhetoric, and new ideas about algebra and its role in rational thinking appeared.

The second result is an understanding of the similarities between the two authors, in particular the fact that they both took their matter from the Italian culture of the time preceding them. This is true not only for the mathematics of Cardano and Tartaglia, but also for the complex context in which they inserted this art while transforming it into a discipline or a science.

So far, the culture surrounding French algebra of the sixteenth century could only be described by referring to an all encompassing Ramism: This new picture of the French algebraic tradition justifies it on its own grounds, making clearer the origins of the algebraic program as independent from Ramus, and its development within a context in which the influence of Ramus is only one among others.

In the second phase, in particular, at the time when Viète was in Paris, some aspects of the juridical culture developed in France had some influence on algebra. First of all, algebra found its own classical origins, and secondly a new position in the classification of mathematical sciences. The first point evolved into the study of the most recently rediscovered source: Diophantus. The denial of the Arabic origins of algebra is typical of the late French tradition, and is easily compatible with the theories of national language and national history flourishing in France in the same milieu.

As to the changing role in the *quadrivium*, algebra managed to move from the lowest part of commercial arithmetic to the role of universal key, capable of providing solutions to problems from all sciences. No longer was its commercial character evident, because its rules were not related to commercial cases, and the problems proposed ceased to have even the slightest relation to practice. Far from being considered *ad hoc* solutions, of which one could not know the truth because of a lack of actual demonstration, algebraic equations became *explanations* in all mathematical sciences. This was a mathematics of the learned people: Guillaume Gosselin, the first author taking Diophantus into account in an algebraic treatise, is the main representative of this phase. He introduces a classification of equations, a notion of question, or abstract algebraic problem, as well as a more developed notation for equations in several unknowns.

As to the legacy of these two authors, it suffices to say that Peletier has been in recent years recognised among Descartes' sources for algebra and algebraic notation, and that Guillaume Gosselin was mentioned, already in the seventeenth century, as an important algebraic author by Salvatore Grisio and other Italian mathematicians. It is especially worth noting here that I have discovered a copy of Gosselin's book annotated by Leibniz.

A first goal for future research, as a result of this new picture would be to answer the question of what made the late sixteenth-century French jurists committed to supporting and developing an algebraic way of thinking, stating and solving scientific questions. This would require the exploration of the connection between algebraic thinking and juridical culture in the late XVIth century in both directions: for not only did the jurists constitute the patronage for algebraic research and most of the readership for mathematical books, but the main figures in the history of algebra were trained in law and some of them practiced law. I have found some texts which theorized the connection between algebra and the rhetoric of the time. The point would be to see to what extent the authors actually practiced this connection, either by using algebra in rhetoric or by using rhetoric in algebra, for they claimed that not only there was a use for algebra in juridical rhetoric, but also that there was a use for rhetorical thinking in sixteenth-century algebra.

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# **Appendix:**

## **QUAESTIO SIVE AEQUATIO: The notion of problem in Descartes' Regulae**

A lot has been thought and written, even recently, about the relationship between the project of a general algebra and "the method" as well as the relationship between this project and the Mathesis Universalis to which Descartes refers in the fourth *Regula*. Though widely discussed, the question of the mathematical sources and, in particular, that of Descartes' scientific education before the draft of the Regulae, remains open. I want to examine here another aspect of Descartes' argumentation, which also concerns the history of mathematical thought, that is the transformation of the idea of problem, by which the scientific problem, in its general form, is seen as an equation and takes on its structure.[276]

We shall here investigate three particular aspects of this transformation: the use made by Descartes, in the Regulae, of the three synonyms of "problem", problema, quaestio, and difficultas, some important aspects of the sixteenth-century tradition of philosophy of mathematics, and finally a preliminary repertory of references to the algebraic manuals of the sixteenth century.

We may well ask whether it is legitmate to subsume under one heading a discussion of these three terms which have been chosen, unified as they are by the meaning we attribute to the term problem. In fact, the three synonyms of problem used by Descartes, problema, quaestio and difficultas, are already interconnected in the text and the connection is already established in both the philosophical and the algebraic traditions. This approach is further justified, precisely by a comparison of the Cartesian text with to the algebraic tradition, which shows that the author himself brings about shifts in the meaning and use of the three

276 To my knowledge, in recent literature, only H.W.Arndt, in his Methodo scientifica pertractatum. Mos geometricus und Kalkülbegriff in der philosophischen Theorienbildung des 17. und 18. Jahrhunderts, Berlin 1971 pp.38-49 has explicitly taken into consideration the connection between the structure of equation and the transformation of the idea of problem. However, Arndt deals with the whole of Descartes' mathematical work, hence he does not aim at determining the mathematical tradition to which Descartes referred at the time of the formulation of the Regulae.

synonyms.  On the other hand, an attempt to clarify the meaning of problem in this text opens up new possibilities for reflection on aspects of Cartesian thought which are very different, such as the doubt and the theory of equations.  For both  were conceived in terms of quaestio.[277]

After placing in context the synonyms of problem used by Descartes, and illustrating their use in the texts of sixteenth-century authors, we shall analyse the occurrences of the terms in the Regulae.  We will draw from sixteenth-century sources of algebraic traditions and will conclude with a short and focussed discussion of the debate on the interpretation of the Regulae.

## *1.  The synonyms of problem and their tradition.*

Aristotle took as an object of specific study the notion of problem, by articulating it through the synonyms of próblêma and erôtêma (or erôtêsis), to which he connected the term  zêtoúmenon (or zêtêma), i.e. problem, question, the thing to be sought.

First of all, he stressed that what distinguishes scientific reasoning from other discourse is the careful formulation of what is sought (zêtoúmenon).  According to Aristotle, there are four kinds of things sought, corresponding to four types of knowledge.  We need to determine tò oti, tò dioti, ei esti, ti esti (***Posterior Analytics***, 89 b 24).  This is what can be asked and what can be answered.  That is to say, in any investigation we look for a medium or whether a medium exists.  In order to state a problem correctly, we have to choose the

277 For erôtesis is the logical form of dubitative questioning, as in the case "Is the universe eternal or not?."  In this sense, the doubt and the  scientific problem are of the same species, and consequently, for instance, posing a doubt correctly is as important posing a problem correctly.

genus and the species common to the things sought (Posterior Analytics, 98 a[278]). The form of the problem is "Does this attribute belong to the given genus?" (Topics, 101 b 30), to which form an apophantic answer can be given. In other words, Aristotle adopted the term próblêma from Pythagorean mathematics, in order to caracterize scientific problems in a general sense, and próblêma became therefore what syllogisms deal with (Topics 104 b 17), whether universal or particular. Furthermore, in the ***Topics*** he distinguished between problems and dialectical problems, where the latter are such that it is not possible to show either of the two alternatives, or there is a doubt because there are strong reasons for both possibilities, as in the case "Is the Universe eternal or not?" (Topics 104 b).

Aristotle used the term erôtêma in a parallel way, and book VIII of the ***Topics*** is devoted to the art of formulating questions. Though this art is typical of the dialectician, the philosopher (i.e. the one who devotes himself to a science) must also make use of it for that sort of individually developed dialogue which is the scientific argument or, in any case, teaching. It is also worth making a distinction between questions relative to demonstration, and those relative to probability. Similarly, we must distinguish between the syllogistic question -- any question which, by definition, assumes as a premise one of two terms of a contradiction -- and the scientific question, which assumes a premise specific to a particular science (Posterior Analytics 77 a 36).[279]

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278 "In order to formulate an investigation, one has to choose the dichotomies and the divisions, putting as a basis the genus which is common to all the objects at issue. For instance, if we want to consider the animals, we have to examine which determinations belong to any animal, and once such determinations are assumed, one has to observe which is the first totality, among those subordinate to the genus, and which are the determinations deriving from any object contained in this totality."(Posterior Analytics, my translation)

279 "If a deductive question (erôtêma) is the same as a proposition which states the half of a contradiction, and every science has its assumptions from which the conclusions proper to that science are drawn; then there must be a scientific question which corresponds to the assumptions

These Aristotelian passages were connected in the sixteenth century in order to give a meaning to the term quaestio which, thanks also to the complex medieval tradition, was more and more the Latin equivalent of probléma. In particular, the passage of Posterior Analytics regarding the zêtoùmena was regularly juxtaposed to the passages of the Topics concerning problémata. Other texts further complicating the use of these synonyms and pointing up the "unstable" meaning of the term próblêma included new editions and translations of Problemata of Aristotle, as well as the increasingly common manuals of the genre erôtémata, which were written in the form of dialogues between master and disciple.

The two Aristotelian works most important for this discussion are, as we have seen, the Posterior Analytics and Topics. In the sixteenth century these two works were put at the center of the reflection on teaching and on the scientific method, often with a polemical choice of emphasis. In particular, those who wanted to maintain the Aristotelian demonstrative idea gave more importance to the Posterior Analytics, whereas those such as Ramus, who were favorable to a rhetorical interpretation of logic and to a specificity of mathematics with respect to logic, privileged the Topics in the definition of logic and of method. This is reflected in the notion of problem insofar as it is only in the Topics that we find the sort of dialectical problem that could be solved in terms of medieval probability. Consequently, the wider reelaboration of this notion, which owes much to the medieval tradition,[280] was furthered by the contributions of the sixteenth-century debate and the

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from which the conclusions proper to that science are drawn."(my translation)

280 There is of course a vast medieval tradition of translations of these Aristotelian passages, and we limit ourselves to mentioning two examples which indicate a hesitation in the translation. As to the passages of the Topics already quoted, while Boethius translates prôblêma with problema, the Translatio anonyma of the XIIth century mentioned in Aristoteles latinus translates próblêma with quaestio. By analogy, for another relevant passage, the one in Posterior Analytics 98 a, we find quaestio in Gerardo from Cremona and problema in Wilhelm of Moerbecke. On these bases, Duns

introduction of the recently rediscovered point of view of Proclus.[281]

We have to consider therefore two main traditions of reference in this, as in many other sixteenth-century debates in the philosophy of mathematics: Aristotle and the Aristotelianism on the one hand, and Proclus on the other.

Obviously, this does not mean that the two traditions developed separately. On the contrary they tended to appear concurrently in the same author, particularly since the Aristotelian tradition is concerned most specifically with logic, and that of Proclus with mathematics. Since they dealt with two different fields, they were compatible. We shall pick up again some points which follow the texts of sixteenth-century authors who dealt with the notion of problem. Then we shall take into brief consideration the meaning of the three synonyms in the thesauri of Budé and Etienne, in the scholae mathematicae of Ramus, and finally in the Conimbricensis and Clavius, insofar as they were part of the teaching received by Descartes.[282]

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Scotus defines problema and quaestio in the same way: each of them supposes something certain and searches something unknown. Furthermore, at least from Abelard, on the logic and philosophical discussion had taken the form of quaestio, which included the statement, the arguments *pro*, the arguments *contra*, the conclusion of the author and the confutation by points (the difficultates).

281 For the history of XVIth century editions of the Commentary to the I book of Euclid's Elements, see for instance G. Crapulli, Mathesis universalis, Roma 1969.

282 Descartes' explicit reference to the Conimbricenses is in AT III 185, 11-12, together with the one to Rubius. We know from François de Dainville that Clavius' text was used at La Flèche. See "L'enseignement des mathématiques dans les Collèges Jésuites de France du XVIe au XVIIIe siècle", Revue d'histoire des sciences 7 (1954), 6-21, 1O9-23. It is also known that Suarez, Fonseca and Toletus were studied there, although this lies outside the scope of the present study. Suffice it to say that Fonseca (Institutionum dialecticarum libri octo) had suggested a distinction adopted by Descartes, though with another meaning, between perfect and imperfect questions (quaestio quae perfecte intelligitur). See J.Sirven, Les années d'apprentissage de Descartes, Paris 1928, p.405, footnote 5; p.406, footnote 1, and also L.J. Beck, The method of Descartes: a study of the Regulae (Oxford 1952). For XVIth century Aristotelianisms and their relation with Descartes, see the classic

Budé writes: "Próblêma est quaestio, id est zêtêma.[283] He explicitly identifies the meaning of the three terms used by Aristotle. Budé picks up again a statement of Aristotle concerning problems:

> Próblêma kaì prótasin idem esse Aristoteles docet in I Topicôn, differentiamque esse tantum in modo enunciandi

In the Topics 101 b 11, Aristotle states precisely that the foundations of discourses (lógoi) and the arguments of reasoning (syllogismoi) coincide, for the foundations of discourses are propositions (protáseis), whereas the arguments of reasonings are problems (*problemata*).

Budé, however, does not distinguish, as does Aristotle, between the notion of problem in general and the notion of dialectical problem. He instead picks up the distinction, taken from Proclus, and typical of mathematics, between problems and theorems. Proclus argued that while theorems are propositions concerning the nature of figures (thus distinguished in statement, data, definition or conditions of possibility, construction, demonstration and conclusion), problems are propositions concerning operations on figures, such as construction, addition, substraction and division.

Budé takes up this contrast by explaining that a problem proposes to find something, and a theorem to teach something, concerning the nature of the object. The rest of the passage contains references to Alexander of Aphrodisias, Themistius, Ammonius and Cicero, all concerning the notion of dialectical problem and providing a number of

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works by J.H.Randall, Wilhelm Risse, Charles Schmitt (e.g. Aristotle in the Renaissance, Cambridge 1983).

283 See Budaeus, Commentarii linguae graecae, in particular the Parisian edition of 1548, which presents the discussion on Proclus, as opposed to the one of 153O. This is not surprising, since Proclus' text was published only in 153O, in Basel. Próblêma is dealt with from p.46O to p.462.

examples. Henri Etienne[284] cites Budé generously, and adds references to Aristotle and Proclus. An aspect of the treatment by Proclus taken up again by sixteenth-century authors, is the "history" of the role of this notion among the Greek mathematicians. He writes that the notion of problem was the object of disputes within the Platonic school. Among Platonist followers of Speusippus, mathematical deductions were seen exclusively as theorems and not as problems, while in the school of Menecmus, mathematical deductions had to be considered only as problems, insofar as they refer to a construction. Proclus noticed also that two other theses, that of Carpus, for the priority of problems with respect to order, and that of Geminus, for the priority of theorems with respect to dignity, were compatible and complementary.

These points were taken up again by Ramus in the Scholae Mathematicae, where he dealt with questions of philosophy of mathematics arising from the rediscovery of Proclus and the other classics. Ramus' goal was to show the expository obscurity of mathematical tradition. He proposed to reorganize the whole of mathematics, taking into consideration the reciprocal articulation of the method of teaching and that of discovery. In this context, he dealt with Proclus' classification of problems, based on the number of solutions, which will remain an alternative to the more famous one provided by Pappus in the Collectiones Mathematicae, according to which geometrical problems are plain, solid or and linear.[285] In this way, Ramus took up again the valorisation of Greek geometrical analysis in a manner typical of the sixteenth century, where analysis was used in two senses: the demonstrative procedure "inverse" to the synthesis, and also that of the "treasure" of analysis, that is the

284 Stephani, Thesaurus graecae linguae, Parisiis 1572.

285 See Pappus Collectio mathematica liber III 2O-22; liber IV 57-59.

corpus of geometrical treatises, oriented toward the solution of problems and particularly toward the transformation of problems in order to make them solvable.[286] Analysis contributed to the emergence of the heuristic aspect of mathematics and to the definition of the activity of mathematicians as a systematic resolution of problems, or as research into the methods of solution. Ramus did not go along with this development, since he preferred to foreground the need for a rigorous and pedagogically effective presentation. He nonetheless tended to take for granted the conception of mathematics, spread by the rediscovery of analysis, as more a set of problems than a set of theorems, and of the mathematical tradition as a set of theorems useful for solving problems. This view, which would be taken up again widely in the XVIIth century,[287] made it necessary to go back to ancient authors, insofar as they maintained the thesis that all mathematical propositions have to be considered problems.

Ramus therefore avoided the conciliatory conclusion of Proclus, and even polemicized against him, stressing that the distinction between problems and theorems was nothing but a scholastic exaggeration. We are here concerned with precisely that question of style that most preoccupied the philosophers of mathematics: i.e. the compatibility of the model of rigor present in Euclid with that proposed by Aristotle.

As Ramus makes clear in his text, Proclus had maintained that Euclid made use of all the kinds of quaestio: "Quaerit enim an est, quid est, quale quid est, propter quid est." To which Ramus answers:

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286 See M.S.Mahoney, "Another look at Greek geometrical analysis", Archive for the history of exact sciences, 5 (1968/69).

287 One should recall, for instance, the concluding passage in Computatio sive logica by Hobbes, drawing the distinction between theorem and problem.

> At, Procle diligentissime, Euclides nusquam quaerit an sit, aut quid sit linea, superficies et corpus, sed sine quaestione docet et definit: problemata quidem quaestiones quaedam videntur esse, quomodo fabrica constituenda sit, sed vanitas ista mox apparebit: et tamen problemata ista affirmant non dubitant.[288]

Proclus furthermore had stressed an aspect of the problems (precisely those posed by Eucldean problems) picked up again by Ramus, which would have a great impact on the XVIIth century algebrists and Descartes, that is the idea of problem as constitution by a given thing (tò dedómenon) and something solved (tò zêtoúmenon).

Clavius dealt with the distinction between theorem and problem, in his In disciplinas mathematicas prolegomena, an introduction to the course in mathematics devoted to the work of Euclid, which is found in the first volume of his Opera.[289] Clavius, first of all, refers to the classical distinction: there are in mathematics "problems" in which we try to build, and "theorems" in which we look for the quality of something. However, in constrast to Proclus, both are meant first as types of demonstrations rather than types of propositions. In fact the passage has a logical weight not only in the broad Aristotelian sense, but also in a more restricted sense, for it refers to the discussion de certitudine mathematicarum which was taking place at that time.[290]

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288 Petri Rami, Scholarum mathematicarum libri, Basileae 1569, p.83.

289 Cristophori Clavii Bambergensis Operum mathematicorum tomus primus, Moguntiae 1611, p.8.

290 We can recall only the main points of this debate, even though it was a very important reference for the three authors. Piccolomini maintains, against Proclus, that mathematical demonstrations do not explain causes, and in this sense they do not follow the Aristotelian ideal. This theory is in a strict connection with the notion of quaestio, because Piccolomini maintains that mathematics is not able to answer the question of "propter quid", among the various types of question. Certainty in mathematical demonstrations was maintained by Averroes, not by Aristotle, and it is reaffirmed by Piccolomini insofar as it is guaranteed by the fact that mathematical objects are obtained by abstraction. Besides Alessandro Piccolomini, Commentarium de certitudine mathematicarum disciplinarum, Romae 1547, the main texts are Francesco Barozzi, Opusculum, in

There are two contexts in which we talk about problema according to Clavius: the context of the mathematicians, who follow the given definition, and the context of the dialecticians, who call problema that quaestio of which both sides are probable. But, Clavius says, we must make a great distinction between the dialectical problem and the mathematical problem. One leads to probability, the other to certainty.

In concluding his argument, Clavius again makes the distinction:

> Itaque ut uno verbo dicam, quaesitum illud Mathematicum construere aliquid docens, cuius etiam oppositum potest effici, Problema; illud vero, quod nihil docet construere, et cuius pars opposita perpetuo falsa existit, Theorema appellatur.(op.cit. p.8)

The two terms here indicate therefore, not so much two propositions as in Proclus, or two demonstrations, as Clavius suggested at the beginning of the passage, but precisely two quaesita. This appears to be a concession to the thesis that mathematics is constituted of problems, even though we must distinguish between problems dealing with construction and those dealing with demonstration. It is interesting also to note that, by referring only to the dialectical problem, Clavius omits mention of the meaning of the scientific Aristotelian problem. Rather, he replaces it simply by the mathematical problem, the opposite of which is always false. It is a legitimate assimilation if we interpret it in the light of the already mentioned Aristotelian thesis about scientific questions (Posterior Analytics 77 to 36).

The sixteenth-century Aristotelian commentaries also had another reason to stress the notion of problem: they were not only looking for a model of rigor or for a way to

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quo una Oratio et duae Quaestiones: altera de certitudine, et altera de medietate Mathematicarum continentur, Patavia 156O; the Commentarii Collegii Conimbricensis; and Giuseppe Biancani (a student of Clavius), De mathematicarum natura dissertatio, Bologna 1615. On this topic, see Giovanni Crapulli, op. cit. 1969, chap.II and Peter Dear in Mersenne and the learning of the schools, Ithaca 1988, cap.IV. See also the various essays in Aristotelismo veneto e scienza moderna (Atti del Centro per la tradizione aristotelica nel Veneto, a cura di L.Olivieri), Padova 1983.

distinguish between theorems and problems, but they also particularly needed to turn again to the search for media for syllogisms, i.e. to the inventio, and its importance in Aristotle (specifically in the second book of the Posterior Analytics, which deals with the search for media, and in the Topics). These two themes are obviously connected, since the ascendent motion of the inventio cannot, by itself, lead to the rigor of the syllogism of the first figure, but only to probability, to the plausible.[291] As Rubius[292] writes:

> Propria inveniendi via est quaestio, vel interrogatio: merito ergo interrogationum, vel quaestionum numerus primo loco ponitur, ut viam teneamus, per quam medium invenire possumus.

The Conimbricenses,[293] in particular in the commentary on the Topics, collect the positions of previous commentators, for instance on the interpretation of the passage in which Aristotle states that the number of problems is the same as the number of protàseis, that is, propositiones. Moreover, they pick up again the distinction of three forms of questions, or problems, classified also by discipline:

> tres partes consequentes problema dividunt in tres quasi species; quaedam enim problemata dicuntur moralia, quia ad scientias practicas conducunt; quaedam pertinent ad Theoreticas, Physicam, Metaphysicam, et Mathematicas; Alia denique sunt logica, quae propterea adminiculantia, id est, opitulantia vocantur.(p.749)

The Conimbricenses intervened also in the dispute De certitudine mathematicarum, taking a

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291 See, besides the classic studies on humanism, also the more recent article by Lisa Jardine, "Lorenzo Valla: academic scepticism and the new humanist dialectic", in The skeptical tradition, ed. M. Burnyeat, Berkeley 1983. Ramus devoted to quaestio a whole chapter of his Aristotelicae Animadversiones of 1548. Already in the 1543 edition quaestio was treated as a correct formulation of the doubt giving rise to the inventio.

292 Antonio Rubio was mentioned by Descartes as among the authors he studied at La Flèche. The quotation is from p.695 of Logicae mexicanae sive commentarii in universam Aristotelis logicam, Coloniae Agrippinae 16O5.

293 Commentarii Collegii Conimbricensis e Societate Iesu: In universam dialecticam Aristotelis Stagiritae, Coloniae, 16O7.

position similar to that of Piccolomini. Although they did not attribute to the mathematical demonstrations causal explanations (that is the perfect kind of certainty), they continued however to consider mathematics among the sciences insofar as its problems are theoretical.

## *2. Cartesian usages: a lexical study.*

Now let us consider the use of these terms by Descartes, using the Index des Regulae.[294]

### 2.a. **Problema**

The text of the Regulae contains four occurrences of the term problema and we will quote fully the relevant passages. The first of them states the goal of the Regulae: to prepare the ingenium to solve all problems. As to demonstrationes, we can recognize also the attribution of a superior value to problems than to theorems. This, as we mentioned earlier, was typical of the revival of Greek analysis, but, also of the algebraic tradition:

> neque enim unquam, verbi gratia, Mathematici evaderemus, licet omnes aliorum demonstrationes memoria teneamus, nisi simus etiam ingenio apti ad quaecumque problemata resolvenda (Regula VII, Crapulli [295] 7,20; AT 367,16)

The fact that Descartes had in mind this specific meaning of problem, which comes from Greek analysis, is shown by the passage in which the word problema next occurs:

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294 J.R.Armogathe, J.L.Marion, Index des regulae ad directionem ingenii de René Descartes avec des listes de leçons et conjectures établies par G.Crapulli, Roma 1976.

295 For each citation, the first page reference is to René Descartes, Regulae ad directionem ingenii, texte critique établi par Giovanni Crapulli avec la version hollandaise du XVIIe siècle, La Haye 1966, and the second to the classic edition by C. Adam and P. Tannery, Oeuvres de Descartes, publiées par Ch.Adam et P.Tannery, Nouvelle présentation, en co-édition avec le Centre National de la Recherche Scientifique, Paris 1964-1974, vol. X.

> satis enim advertimus veteres Geometras analysi quadam usos fuisse, quam ad omnium problematum resolutionem extendebant, licet eamdem posteris inviderint.(Regula IV, Crapulli 12,2; AT 373,12)

However, Descartes had decided not to limit himself to strictly mathematical problems, which he considered only simple examples of scientific problems:

> Neque enim magni facerem has regulas, si non sufficerent nisi ad inania problemata resolvenda, quibus Logistae vel Geometrae otiosi ludere consueverunt; sic enim me nihil aliud praestitisse crederem, quam quod fortasse subtilius nugarer quam caeteri.(ibidem)

The other occurrence is in Regula XIV, in a passage clarifying the relationship between arithmetical and geometrical problems. Here, Descartes indicates that rules are oriented to a more important knowledge, and therefore that mathematical problems must be studied as propedeutical to this knowledge. In this passage, the term quaestio is also used as a direct synonym of problema:

> Optaremus hoc in loco lectorem nancisci Arithmeticae et Geometriae studiis propensum, etiamsi in iisdem nondum versatum esse malim, quam vulgari more eruditum: usus enim regularum, quas hic tradam, in illis addiscendis, ad quod omnino sufficit, longe facilior est, quam in ullo alio genere quaestionum; huiusque utilitas est tanta ad altiorem sapientiam consequendam, ut non verear dicere, hanc partem nostrae methodi non propter mathematica problemata fuisse inventam, sed potius haec fere tantum huius excolendae gratia esse addiscenda.(Regula XIV, Crapulli 63,5; AT 442,5)

In conclusion, problema is used by Descartes always in connection with Logistic. If this term reminds us, first of all, of the Logistica Speciosa that Viète introduced in his In artem analyticen isagoge (Tours, J.Mettayer 1591), the discipline of which Descartes is thinking has a long history, which can be reconstructed in connection with the expanding production of algebra texts by French academics in the second half of the sixteenth century.

Though very little is known of the diffusion of Viète's work before the publication of versions and reductions in the Thirties, it seems that its impact at this time was not sufficient to transform the agenda of the discipline. Moreover, there are no documents showing knowledge of Viète by Descartes prior to those years.

One could suppose that Descartes, having learned algebra from both the Parisians and Clavius, heard about Viète[296] and tried to construct the core of his new algebra[297] before he actually read his work.[298] The fact that Descartes could not rely on a previous author for his "symbolic algebra" is clear from his insistence on the substitution of letters for numbers as a way in which he distances himself from the logistic tradition (Regula IV, Crapulli 14,27; AT 377,3).

This appears not only in the "archaic" Regula IV,[299] but more clearly in Regula XVI,

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296 Besides the Isagoge, other works by Viète had been published: these involved not only the application of algebra to geometrical problems, but also a very developed use of the theory of equations. In 1624 there also appeared De aequationum recognitione et emendatione tractatus duo. However, the diffusion of Viète's work is still an open question, with the result that we do not know when it reached even the Parisian mathematicians whom Descartes met before writing the Regulae (between 1625 and 1628). We know that a member of this group, Pierre Hérigone, published a text which included Viète's algebra only in 1642, i.e. after the most important publications on Viète (translations and reductions) of the early Thirties.

297 Thereby following his own advice to make one's own mind "sagax", as in Regula X.

298 In the first volume of AT, Correspondance, p.477, we find a letter by Descartes to Mersenne dated "fin décembre" 1637 in which Descartes defends the originality of his Géométrie with respect to Viète's work: here Descartes mentions De emendatione aequationum. This letter by Descartes is situated at the beginning of the polemics with Beaugrand (1638) on the accusation of plagiarism from Viète: Descartes maintains here that he has started where Viète had stopped, specifying that in that moment he was reading Viète more than at any other time before, in order to check to what extent the accusations were justified: "Et ainsi i'ay commencé où il avait acheué; ce que j'ay fait toutesfois sans y penser, car i'ay plus feüilleté Viete depuis que i'ay receu vostre derniere, que ie n'auois iamais fait auparavant, l'ayant trouué icy par hazard entre les mains d'vn de mes amis; & entre nous ie ne trouve pas qu'il en ait tant sceu que ie pensois, nonobstant qu'il fust fort habile." (AT, Correspondance, vol I, pp.279-28O. See also Descartes' letter to Mersenne of 2O/2/1639), in AT, Correspondance, vol. II.

where we have a clear indication of what Descartes meant by Logistica. He writes:

> primo advertendum est, Logistas consuevisse singulas magnitudines per plures unitates, sive per aliquem numerum designare, nos autem hoc in loco non minus abstrahere ab ipsis numeris, quam paulo ante a figuris Geometricis, vel quavis alia re. Quod agimus, tum ut longae et superfluae supputationis taedium vitemus, tum praecipue, ut partes subiecti, quae ad difficultatis naturam pertinent, maneant semper distinctae, neque numeris inutilibus involvantur: ut si quaeratur basis trianguli rectanguli, cuius latera data sint 9 & 12, dicet Logista illam esse V225 vel 15; nos vero pro 9 & 12 ponemus a & b, inveniemusque basim esse Va + b , manebuntque distinctae illae duaes partes a & b , quae in numero sunt confusae. (Crapulli 73,11-23; AT 455-456)

Without entering here into a detailed discussion of Desartes' choice of notation,[300] we limit ourselves to the observation that his system, while borrowing some elements from the French tradition, from Scheubel to Viète, nonetheless constitutes both a critique and a modification of that tradition. The term logistica, already present in Plato,[301] was applied in the sixteenth century to algebra in order to stress the fact that this art belonged to the classic tradition. It is used in this sense in the homonymous work of Jean Borrel, and also in the Lexicon by Dasypodius:

> Logistica est scientia, aut contemplatio numerorum denominatorum. (...) Logistica quoque dividitur in supputationem quae fit compositione et alteram

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299 Schuster has shown that Regula IV belongs to the first draft of the Regulae (1619), immediately followed by the composition of the first part of the treatise, up to Regula XI. The other *regulae* must have been composed between 1626 and 1628. They reflect a more mature experience of scientific problems within Mersenne's circle, and they share with the latter an apologetic aim. See John Schuster "Descartes' mathesis universalis: 1619-28", in S. Gaukroger, Descartes. Philosophy, Mathematics and Physics, Brighton 198O.

300 See on this note 6, by Pierre Costabel, to the Regula XVI, in: René Descartes, Regles utiles et claires pour la direction de l'esprit en la recherche de la vérité, par Jean-Luc Marion, La Haye 1977.

301 See Jacob Klein, "Die griechische Logistik und die Entstehung der Algebra", Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik, Abt.B: Studien, Vol. 3, fasc. 1 (Berlin, 1934), fasc. 2 (1936), English trans. Greek mathematical thought and the origin of algebra, Cambridge MA 1966.

quae fit resolutione.[302]

In conclusion, by Logistica we should understand French algebra of the second half of the sixteenth century, as developed especially but not exclusively, by Viète, who summarized his program with the famous sentence "Nullum non problema solvere." As for this specific topos of the Logistica tradition, a topos which has been invoked a few times in the Regulae, I will mention only that Cardano[303] already gives a version of it and that this same phrase of Viète's is taken up again by Van Schooten.[304] Descartes himself ironizes Viète's use of this topos in a letter to Mersenne in 1632.[305]

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302 Conradus Dasypodius, Lexicon mathematicum, Argentorati 1579, p. 1.

303 Girolamo Cardano, Ars Magna, chap. 1. See for instance Hieronymi Cardani Mediolanensis philosophi ac medici celeberrimi operum tomus quartus, quo continentur arithmetica, geometrica, musica, Lugduni, J. A. Huguetan & M. A. Ravaud, 1663, p. 222: "Cum omnem humanam subtilitatem, omnis ingenii mortalis ars haec superet, donum profecto coeleste, experimentum autem virtutis animorum, atque adeo illistre (sic), ut qui hanc attigerit, nihil non intelligere posse se credat."

304 See in particular the epilogue to the edition of Viète's works, (Francisci Vietae, Opera mathematica. In unum volumen congesta, atque recognita. Opera atque studio Francisci à Schooten Leidensis, Matheseos Professoris. Lugduni Batavorum, Ex Officina Bonaventurae et Abrahami Elzeviriorum, 1646) p.545-546 In Isagogen. Here Schooten explains that the motto "Nullum non problema solvere" should be understood as "Omne in quo de quantitatum aequalitate vel proportione inquiritur, problema utcunque solvere." The theory of equations in Descartes' Géométrie gave a new sense to the motto that Viète had formulated in connection with his own theory of equations.

305 This is the letter dated 3/5/1632: "Je vous remercie du livre d'Analyse que vous m'avez envoyé; mais entre nous, ie ne vois pas qu'il soit de grande utilité, ny que personne puisse apprendre en le lisant la façon, ie ne dis pas de nullum non problema solvere, mais de soudre aucun probleme, tant puisse -t-il estre facile. (...) le problème de Pappus, car il faut bien aller au delà des sections coniques et des lieux solides, pour le resoudre en tout nombre de lignes données, aussi que le doit resoudre un homme qui se vante de nullum non problema solvere, & que ie pense l'avoir resolu."(AT I,244). According to the editors, the irony is directed to Beaugrand, and the book of analysis is his translation and commentary of Viète's Isagoge (Paris 1631). A comparison between some aspects of Viète's work and some passages of the Regulae has been done recently by M. Tamborini, "Tematiche algebriche vietiane nelle 'Regulae' e nel primo libro della 'Géométrie' di Descartes, in Miscellanea secentesca. Saggi su Descartes, Fabri, White, Milano 1987.

2.b. **Quaestio**

The structure of the Regulae ad directionem ingenii establishes that the first twelve Regulae concern simple propositions and the others (of which there were meant to have been twenty-four) deal with the quaestiones. First of all, we should understand by quaestio a proposition which is the composition of simple propositions and expresses the composition of simple ideas. The first part should therefore be considered propaedeutical to the other two, relative to questions. The second of the two latter ones, which were to have taken into consideration questions not immediately reducible to algebraic or geometrical questions, was never written. Nevertheless, in the two existing parts, the notion of quaestio has great importance. Since our goal here is to argue the thesis that the notion of quaestio is reestablished and put at the center of the reflection of the Regulae, it is worth recalling that, according to the Index des Regulae the text contains 90 occurrences of the verb quaero and 58 occurrences of quaestio: they are obviously among the most frequently used verbs and nouns in the whole work. The frequency of use of these terms argues for the prominence of this theme, despite its neglect by previous scholars. Before going on to categorize the various meanings of the term quaestio, we will examine the passage in which Descartes gives a complete example of "perfectly understood quaestio", that is of the kind of question which can most easily be reduced to quaestiones perfectae of arithmetic and geometry.[306]

Descartes states that in any question, whether or not it is perfectly understood,[307]

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306 Beck has analyzed this passage, in connection with another example taken from the Dioptrique, in the chapter "The solution of problems" of his work, already cited. It is, however, worth re-examining in this different context.

307 See. Regula XIII, Crapulli p.54, AT 430,17.

1)there is something unknown;

2)that something unknown must be designated in some way;

3)it must be designated by something known.

Furthermore, a perfectly understood question is more determined:[308] for we know distinctly from what to distinguish the unknown thing, what is sufficient to find it and, finally, in what way we can show the interdependence between the two things. Descartes remarks that the first two are features common also to imperfect questions, as when we ask in abstract terms, "What is the nature of the magnet?" In order for the question to be perfectly understood, it must be formulated in such a way that everything is included in the data. The question well put will therefore be: "What can we state about the nature of the magnet starting from the experiments of Gilbert, true or false as they might be?" Interestingly, Descartes himself explains that the question is not limited by experimental data, but rather consists in establishing the conditions of possibility for a solution. One might draw an analogy to the diorismòs of Euclid's theorems.[309] Here the reduction of a question to the equation between the unknown and an appropriate relation among known quantities, appears not so much as a technical or mathematical issue. Rather the theorization already present in algebra is held up as a preferable alternative to syllogistic dialectic. In other words, also in connection with the

308 "Notandum est, inter quaestiones quae perfecte intelliguntur, nos illas tantum ponere, in quibus tria distincte percipimus: nempe, quibus signis id quod quaeritur possit agnosci, cum accurret; quid sit praecise, ex quo illud deducere debeamus; et quomodo probandum sit, illa ab invicem ita pendere, ut unum nulla ratione possit mutari, alio immutato." (*Regula* XII, Crapulli p.73, AT p. 429, 13).

309 Oo *diorismoi,* see Mahoney (1968-69).

passages relative to problema already examined, I want to argue that what are considered Descartes' most relevant contributions to algebra,[310] which could be characterized as the foundation of an algebra of geometry, do not give an adequate account of the importance of the role of algebra in his thought. Once algebra has replaced Aristotelian logic, it will allow anyone to formulate problems in any science. Descartes, already in the first Regulae, emphasizes that problems of arithmetic and geometry are nothing but a simple example of application of this method. In spite of all the importance attributed by Descartes to the simple, they are nothing but first exercises of the method.[311]

The quaestio is defined in the following way, after more than 40 occurrences of the term:

> Intelligimus autem per quaestiones, illa omnia in quibus reperitur verum vel falsum: quarum diversa genera enumeranda sunt ad determinandum, quid circa unamquamque praestare valeamus.(Regula XIII, Crapulli 56,3; AT 432,13)

This seems to stress the role of the quaestio in a new logic, which is confirmed by the rest of the passage:

> Iamiam diximus, in solo intuitu rerum, sive simplicium, sive copulatarum, falsitatem esse non posse; neque etiam hoc sensu quaestiones appellantur,

310 I.e. the improvement of notation, the overcoming of the principle of homogeneity through the definition of the unit, and the reduction of geometrical to algebraic problems through the reduction of any quantity to length.

311 In addition to the previous quotation and the remarks about "problem", we may cite the conclusion of Regula XII, after the definition of perfectly understood question: "Cujusmodi quaestiones, quia abstractae sunt ut plurimum, et fere tantum in Arithmeticis vel Geometricis occurrunt, parum utiles videbuntur imperitis; moneo tamen in hac arte addiscenda diutius versari debere et exercere illos, qui posteriorem hujus methodi partem, in qua de alijs omnibus tractamus, perfecte cupiant possidere." This stresses that the complete treatise is oriented towards the last twelve rules, devoted to the imperfectly understood questions, which belong first of all to mathematics but not to arithmetic or geometry, i.e. to "scientiae mediae", or even natural philosophy.

> sed nomen istud acquirunt, statim atque de iisdem iudicium aliquod determinatum ferre deliberamus.

For the subject here is logic, insofar as Descartes divides all knowledge into *propositiones simplices* and quaestiones.

> Caeterum, ne quem forte lateat praeceptorum nostrorum catenatio, dividimus quidquid cognosci potest in propositiones simplices et quaestiones. Ad propositiones simplices non alia praecepta tradimus, quam quae vim cognoscendi praeparant ad obiecta quaevis distinctius intuenda et sagacius perscrutanda, quoniam hae sponte occurrere debent, nec quaeri possunt; quod in duodecim prioribus praeceptis complexi sumus, ac in quibus nos ea omnia exhibuisse existimamus, quae rationis usum aliquomodo faciliorem reddere posse arbitramur.(Regula XII, Crapulli 53,9; AT 428,21)

The question perfectly understood is, in other words, that in which the relationship or rather the composition among simple propositions is distinct and determined, and the case of the nature of the magnet is very much to the point. Descartes' thinking about this matter is revealed by a previous passage in Regula XII, which is fundamental for our argument, where, among other things, difficultas is used as an alternative to quaestio:

> Colligitur tertio, omnem humanam scientiam in hoc uno consistere, ut distincte videamus, quomodo naturae illae simplices ad compositionem aliarum rerum simul concurrant. Quod perutile est annotare; nam quoties aliqua difficultas examinanda proponitur, fere omnes haerent in limine, incerti quibus cogitationibus mentem debeant praebere, et rati quaerendum esse novum aliquod genus entis sibi prius ignotum:(Regula XII, Crapulli 52,3; AT 427,3)[312]

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312 The text continues, "Ut si petatur quid sit magnetis natura, illi protinus, quia rem arduam et difficilem esse augurantur, ab iis omnibus quae evidentia sunt animum removentes, eumdem ad difficillima quaeque convertunt, et vagi expectant utrum forte per inane causarum multarum spatium oberrando aliquid novi sit reperiturus. Sed qui cogitat, nihil in magnete posse cognosci, quod non constet ex simplicibus quibusdam naturis et per se notis, non incertus quid agendum sit, primo diligenter colligit illa omnia quae de hoc lapide habere potest experimenta, ex quibus deinde deducere conatur qualis necessaria sit maturarum simplicium mixtura ad omnes illos, quos in magnete expertus est, effectus producendos; qua semel inventa, audacter potest asserere, se veram percepisse magnetis naturam, quantum ab homine et ex datis experimentis potuit inveniri. Denique colligitur quarto, ex dictis, nullas rerum cognitiones unas aliis obscuriores esse putandas, cum omnes eiusdem sint naturae, et in sola rerum per se notarum compositione consistant.", which I prefer to stress by my paraphrasis.

Here we are offered clarification of what has been already anticipated with respect to Aristotle. Descartes takes from the Philosopher the centrality of the quaestio and stresses it by elaborating an articulated alternative to the classical way of stating the problem scientifically. Descartes writes that the person who asks a question expects to find a genus of entities so far unknown. In this way, when a Dialectician asks himself what is the nature of the magnet, he forgets what is obvious (i.e. "simple") and turns to look for what is really difficult, expecting to find it among the many causes. Descartes proposes, on the contrary, to start from the presupposition that we cannot know anything about the magnet, but an unknown combination of simple natures already known. We ought, therefore, to collect the results of experiments with the magnet, which will point out the effects it provokes and, since these effects can be traced to a certain number of simple causes, we will be able to say that the nature of the magnet is nothing but a composition of those simple natures.

We have seen that Descartes stressed the importance of the themes dealt with in the scientific enterprise, first of all by quaestiones as illa omnia in quibus reperitur verum vel falsum and also in the passages which follow, where there are obvious references to Aristotle, such as quidquid cognosci potest and omnem humanam scientiam. In this way, Descartes indicates his awareness that the upheaval provoked by algebra within scientific procedures concerns not only the possibility of solving all problems, but also of putting them in such a way that they can be solved.[313] It is in this context that we should interpret the reference to comparatio:

> Et quidem omnia haec entia iam nota, qualia sunt extensio, figura, motus, et similia, (...) per eandem ideam in diversis subiectis cognoscuntur (...); haec

313 The passage by Van Schooten already cited should be seen precisely in this light, since to the mathematical heritage of Viète, Van Schooten added the confidence in the extension of the scope of mathematical sciences characteristic of the Cartesian programme.

> idea communis non aliter transfertur ex uno subiecto ad aliud, quam per simplicem comparationem, per quam affirmamus quaesitum esse secundum hoc aut illud simile, vel idem, vel aequale cuidam dato: adeo ut in omni ratiocinatione per comparationem tantum veritatem praecise agnoscamus. (Regula XIV, Crapulli 61, AT p.439,11)

For the conclusion to which Descartes comes in his analysis of the quaestio on the nature of the magnet, is that all and only the human knowledge of nature, for instance, of the magnet, is in the comparatio. This is in fact the result of the formulation of the quaestio, first by its decomposition and recomposition in terms of an equation, as indicated earlier, and then in the possibility of substitution, which is induced by the theory of equation. For if the initial equation is between the unknown, that is the nature of the magnet, and a composition of simple nature or known things, such an equation authorizes an unlimited number of substitutions. Therefore, any knowledge is determined only within this network of comparison or substitutions.

Descartes stresses further the contrast between this approach and the approach of Dialecticians:

> Sed quia, ut iam saepe monuimus, syllogismorum formae nihil iuvant ad rerum veritatem percipiendam, proderit lectori si, illis plane reiectis, concipiat omnem omnino cognitionem, quae non habetur per simplicem et purum unius rei solitariae intuitum, haberi per comparationem duorum aut plurium inter se. Et quidem tota fere rationis humanae industria in hac operatione praeparanda consistit; quando enim aperta est et simplex, nullo artis adiumento, sed solius naturae lumine est opus ad veritatem, quae per illam habetur, intuendam.(ibidem)

From the definition and determination of the quaestio, he shifts his attention to the preparation of the comparatio: i.e. we understand that the goal of the perfect determination of the quaestio consists in establishing the comparatio. The preparation of the comparatio assumes a fundamental importance in the new scientific procedure, so much so that it motivates the writing of the regulae itself.[314] The regulae most relevant to this problem, however, do not exist. As we have said before, the third book should contain twelve rules concerning quaestiones imperfectly understood, which should then be reduced to the previous ones, as in the case of the nature of the magnet. This is not the place for an extended discussion of the term comparatio, but we should remember that this term could be interpreted by reciprocal connection between the mathematical notion (which is connected to proportion, equality and disequality, parabolism and syncrisis of Viète) and the logical notion.

We conclude by recalling that we have taken into consideration only passages that directly explain the notion of quaestio. The terms occurs 29 times with a specifically Cartesian m1aning, as explained above, and four times times with its generic Latin meaning. Some of the occurrences considered here concern the distinction between direct and indirect questions. This is an explicit reference to the theory of equations, where direct questions take the form of simple proportions, or equation of first degree, whereas the indirect questions involve the determination of a medium proportional, and therefore they involve equations of, at least, second degree.

2.c. **Difficultas**

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314 The use of the term comparatio is, however, limited to only 7 occurrences. Besides the one already considered, the meaning of ***comparatio*** is explained in the passage that identifies ***comparatio*** and ***aequatio***, i.e. the statement of Regula XIX.

For Beck it is already clear[315] that what Descartes calls "difficulté" in the Discours de la méthode (we should think for instance of the second precept) corresponds to what he calls quaestio in the Regulae. We must, however, define the relationship between difficultas and quaestio in the Regulae itself.

The first occurrence stresses the relationship with the scholastic tradition by inviting us to perfect the natural light of reason non ut hanc aut illam scholae difficultatem resolvat (Regula I, Crapulli 3,6; AT 361,19).[316] However, it is the next passage, by summarizing what we have already seen, which most clarifies the Cartesian approach to the notion of problem (to which here Descartes refers only by the term difficultas). It is taken from Regula VI:

> monet enim res per quasdam series posse disponi, non quidem in quantum ad aliquod genus entis referuntur, sicut illas Philosophi in categorias suas diviserunt, sed in quantum unae ex aliis cognosci possunt, ita ut, quoties aliqua difficultas occurrat, statim advertere possimus, utrum profuturum sit aliquas alias prius, et quasnam, et quo ordine perlustrare.(Crapulli 17-18; AT 381, 8)

Among the occurrences which show how Descartes used quaestio and difficultas differently, I will cite only Regula XIII (Crapulli 55,17; AT 431,19), in which we find the phrase difficultatem bene intellectam, an expression used at first, e.g. in the statement of the Regulae itself, only for quaestio. Also relevant in this connection, is Crapulli 21,8 and 25; Regulae VI AT 386,1 and 21, which concerns the application of the notion of quaestio directa or indirecta to the analogous difficultas. The following passages confirm this interpretation: Regula VIII, Crapulli 27,3,6 and 30; AT 393,14 and 17, 394,21; Regula XIII, Crapulli 55,25, AT 432,1; and Crapulli 57,17; AT 434,13; particularly explicit is the use of

315 See L.J.Beck, op. cit., p.2O7, nota 2.

316 I interpret in in this sense also the occurrence in the Regula IV, Crapulli 13, 23; AT 375, 21.

difficultas as synonym of quaestio in the passages of Regulae XIV, Crapulli 62,10; AT 440,24and of Regulae XVII, Crapulli 76,2 and 7; AT 459,11 and 17.[317]

Strictly speaking, however, difficultas is contained in the quaestio of which it is the problematic kernel,[318] as Decartes states in the Regula XIII:

> Quaestione sufficienter intellecta, videndum est praecise, in quo difficultas eius consistat, ut haec ab aliis omnibus abstracta facilius solvatur.
> Non semper sufficit quaestionem intelligere, ad cognoscendum in quo sita sit eius difficultas; sed insuper reflectendum est ad singula quae in illa requiruntur, ut si quae occurrant nobis inventu facilia, illa omittamus, et illis ex propositione sublatis, illud tantum remaneat quod ignoramus. (Crapulli 59,25; AT 437,12)

This particular meaning of difficultas is also clarified in Regula XIV:

> Maneat ergo ratum et fixum, quaestiones perfecte determinatas vix ullam difficultatem continere praeter illam, quae consistit in proportionibus in aequalitates evolvendis; atque illud omne, in quo praecise talis difficultas invenitur, facile posse et debere ab omni alio subiecto separari, ac deinde transferri ad extensionem et figuras, de quibus solis idcirco deinceps usque ad regulam vigesimam quintam, omissa omni alia cogitatione, tractabimus. (Crapulli 62,31; AT 441,21)

A few other passages should be interpreted in this sense: Crapulli 20,25 and 32; AT 385,10 and 17; Crapulli 21,8 and 18; AT 386,1 and 14. An important example of the reduction of a difficulty is the reduction of measurement to order:

> Sciendum etiam, magnitudines continuas beneficio unitatis assumptitiae posse totam interdum ad multitudinem reduci, et semper saltem ex parte; atque multitudinem unitatum posse postea tali ordine disponi, ut difficultas, quae ad mensurae cognitionem pertinebat, tandem a aolius ordinis inspectione dependeat, maximumque in hoc progressu esse artis

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317 See also the passages: Crapulli 21,25-31; AT 386,22-25; Crapulli 27,3 e 6; AT 393,14 e 17; Crapulli 27,28; AT 394,21; Crapulli 55,25; AT 432,1; Crapulli 57,17; AT 434,13; Crapulli 72,25; AT 455,8; Crapulli 76,25 e 77,2; AT 460,12 e 23; Crapulli 81,6 e 28; AT 467,21; 468,23.

318 See also, with the same meaning, the passage at the end of Regula XI: "Ad quae et similia qui reflectere consuevit, quoties novam quaestionem examinat, statim agnoscit, quid in illa pariat difficultatem, et quid sit omnium simplicissimus <solvendi> modus" (Crapulli 39,27; AT 410,11).

adiumentum.(Regula XIV, Crapulli 7O,16; AT 451-452)

At this point, the part treated by the method is the kernel of the quaestio, identifiable with difficultas, so that the statement of the Regula XVI speaks directly of proposita difficultas, and the explanation refers to determinatae difficultates et perfecte intellectae.[319]

Insofar as it is the totally mathematizable kernal of the quaestio, the difficultas can be completely identified with the aequatio, as is stated in Regula XVII itself.

## *3. THE ALGEBRAIC USE OF THE SYNONYMS OF PROBLEM. QUAESTIO AND AEQUATIO*

### a) **Quaestio**

A question is defined in its parts, the known and the unknown, and in their relationships, and this definition is made explicit by a common notation. There are two points to be made here. The first is that Descartes consistently delimits a question in this way, that is to say, he reconstructs it only with reference to itself, to the terms present in itself and to their relationships. The second point is that the language he uses is directly connected with the theorizations of sixteenth-century algebraic manuals, in which quaestio was, by this procedure, reduced to an equation. That Descartes looks for a reduction of the quaestio to the aequatio is clear from the reading of the text of the Regulae, and it has been taken as a given by many authoritative studies of the Cartesian text. By contrast, the source remains to be investigated, and most importantly the identification between the algebraic problem, articulated in quaestio and problema, and aequatio in the sixteenth-century manuals.

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319 Finally, there are only nine occurrences having the meaning of "obstacle", whether theoretical or oratorical.

The first algebraic manuals were, it is known, of practical character, and tended to concentrate on problems and their solutions. For this reason, they were made up of rules for solutions and by examples. This is the case, for instance, with the Summa Arithmeticae (Luca Pacioli, Venezia, P. de Paganinis, 1494), L'Arithmétique (Etienne de La Roche, Lyon, C. Fradin, 1520) and also the Ars Magna (Girolamo Cardano, Nürnberg, J. Petreius, 1545). It should be noticed that, whereas the first dealt mostly with commercial problems, Cardano's text dealt with abstract numerical problems, i.e. with theoretical problems. Stifel, in his Arithmetica integra (Nùrnberg, J. Petreius, 1543) discusses exempla, most of which are commercial, but some of which are also of theoretical character. In this, he is followed by Scheubel (Algebrae compendiosa facilisque descriptio, Paris, G. Cavellat, 1551), who however writes (page 24):

> Sequuntur nunc quaedam aenigmata, seu quaestiones, quorum solutiones tandem hanc aequationem requirunt.

He deals with abstract numerical problems similar to those of Diophantus, and it is interesting that Scheubel, the author of the first book in which algebra was not presented as a complement of arithmetic (as in the text of Etienne de la Roche) published in France, used quaestio in this sense. In 1553 a text by Peletier, l'Algèbre, appears in Poitiers. It is plausible to consider Peletier as a source for Descartes for various reasons. For one thing, Peletier was a figure of great importance, mentioned also by Montaigne, who met him and talks about him in his Essais.[320] Furthermore, his works received wide diffusion, in particular this one -- L'Algébre would be reprinted in 1554 (Paris), in 1560 (in the same year

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320 Cfr. Michel de Montaigne, Les Essais, Publiés d'après l'exemplaire de Bordeaux par Fortunat Strowski (19O6), Hildesheim 1981: I,126; II,324. Only the second reference is actually mathematical. One should notice also that this is one of the very few references to contemporary mathematicians: the other is to Foix de Candalle, the main translator and commentator of Euclid.

as a Latin version, De occulta parte numerorum quam algebram vocant), in 1609 (Lyon) and 1620 (Genève). Finally, it is mentioned a few times by Clavius in his Algebra, and we know that Descartes read Clavius' Algebra. Peletier follows Stifel both for the themes and for the terminology, and therefore talks about exemples, but in respect to the equation he writes:

> L'equacion et l'Extraccion des Racines, sont deux parties de l'Algebre, equelles consiste toute la consommacion de l'Art. Pource, nous le tretterons toutes deus clerement, et au long. (...)
> Equacion donq, est une equalite de valeur, entre nombres diversement enommez. Comme quand nous disons, 1 Ecu valoer 46 Souz (...) E pour ample declaracion nous ferons une Question familiere, qui sera tele.
> Il y a un Nombre duquel la tierce e la quarte partie otees, laissent 1O: Qui est ce Nombre la?
> Premieremant, Il s'entand assez, que les nombres exprimez es Questions sont ceus qui nous guident: e par l'aide desquez nous decouvrons les Nombres inconnuz. Il faut donc en cette Question proposee, que par le moyen de 10, Nombre exprimé, se trouve celui que je demande.(p.22)

As we can see, the link with the merchant's manuals tradition is strong; however, we find some innovations which will be typical of the manuals of the rest of the century, more marked by humanistic university culture. The main innovation is to treat the equation as a theme in itself, to give it explicit definition and to stress its role. At the same time, Peletier deals with "numbers expressed in questions": quaestio is therefore used by Peletier to establish a common term for the "cases" of business arithmetics and for the abstract numerical or geometrical problem. Furthermore, the use of the term in connection with the introduction of the (elementary) theory of equations, indicates a strict relationship between the two notions. The term quaestio was used in place of the term problema by Tartaglia. In his General trattato di numeri a misure (Venezia, C. Troiano dei Navò, 1556-1560), which collected all in one text the rules for the practical applications and a formulation of the algebra which would be appropriate for those familiar with Euclid, he rarely used the term questione and then only as a synonym of problema.

A few years later Jean Borrel published what was to become a fundamental text for algebra in the sixteenth century, his Logistica (Lyon, G. Rouillé, 1559). The problems he deals with are numerical, but they represent classes of problems and are called, in the classical sense, problemata.[321] However, the last chapters are devoted to quaestiones, which are explicitly meant to deal with all sorts of problems. It is interesting to mention the passage that introduces the fourth chapter, dedicated to questions, insofar as it is extremely close to Cartesian terminology:

> Libris superioribus iactis veluti fundamentis, pars operis nunc superest sane pulcherrima, ipsaque subtilitatis exercitatione fructuosa. Ubi logisticae quaestiones, non solis numeris proponuntur, Arithmeticorum instar problematum, sed rebus variis applicantur, quae vel ad usum vitae, vel ad meditationem ingenii, aut ad utraque simul pertineant. Nam et regularum usus, cum earum sedes, vel rei natura, vel arte propositi sunt in occulto, non aliter melius, aut utilius doceri potest, quam ipsa vestigationis varietate multiplici. Magna etiam traditionum particularium copia, una cum ipsis sese quaesiti aperit. Ad haec autem non via Logisticorum trita communiter incedam, qui multitudine quaestionum libros exagerant, eandem saepius speciem, aliis, atque aliis, mercaturis tanquam diversum applicantes.[322]

We notice in Borrel the same criticisms made by the logistici of the second half of the sixteenth century about their predecessors and in particular about the tradition of abacus books, namely the multiplicity of examples, and the lack of method or of uniformity of procedures which allows them to extend the field of application to useful things for life or ad meditationem ingenii.

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321 In those years another text became fundamental: Pedro Nunez's Libro de algebra in arithmetica y geometria, published in Antwerp in 1564, quickly known in France: in it we find 110 "casos de Arithmetica", on numbers. "Casi" was also the term used by the first Italian manuals.

322 Ioannes Buteonis Logistica quae et Arithmetica vulgo dicitur, Lugduni 1559, p. 197.

Quaestio is found, used in this sense, in the translation of Tartaglia's work into French, published by Guillame Gosselin with an extended commentary as L'Arithmétique de Nicolas Tartaglia Brescian, (Paris, G. Beys, 1578). Gosselin, we should remember, belonged to the mathematical Parisian milieu between 1570 and 1585, in which the influence of Ramus was strong and which was known to Viète. Also here problema corresponds to canon or type of equation, whereas quaestio reverts to the more general meaning of problem. Following these two precedents, but more explicitly, is Stevin, who, in the second book of his Arithmétique, uses the term problema to indicate the problems (which represent classes of equations) the form of which is: "given three terms, of different degrees, to find the fourth proportional."

Then, after defining the rule of the false, he deals with questions which should be considered as applications and more complex examples of the rules of solution indicated in the problems. In them we do not look for a demonstration, nor for a geometrical solution to confirm the algebraic solution: we only solve them "par l'algèbre.[323]"

b) **Aequatio**

In the Regulae the term aequatio appears only twice, and both times in the statement of the non-annotated Regulae (Crapulli 82,2 and 6; AT 469,2 and 6). There are, however, three occurrences of aequalitas, the term used, for instance, by Viète, and expecially there is a repeated use of comparatio in an algebraic sense, as establishing an equation. This is not by itself surprising, insofar as comparatio was a mathematical term which indicated

323 See The principal works of Simon Stevin, vol. II B, Mathematics, edited by D.J. Struik, Amsterdam 1958, p.681.

numerical inequality, or the relation, for instance in Ramus,[324] and which was used as an alternative to proportio in the geometrical context. It should also be remembered that the term aequatio, through its variations, did not indicate a state, but preserved the connotation of action, didn't indicate a state, as we have seen in Scheubel's quotation. We may wonder why Descartes avoided the use of the two algebraic terms (aequatio and proportio). My conjecture is that he went backwards along the same logical path that had moved him to adopt the point of view of logistica, and also that he construed a more general philosophical meaning for the idea of equation. What counts here is that this idea is identified not only with quaestio and with difficultas, as we can see from some quotations and from the statement of Regula XVII, but also that it constitutes the presupposition of the idea of problem.

This centrality of equation is the leitmotiv of algebraic manuals which accompanied the constitution of algebra into a discipline in the university and in the academies, and corresponds to the rediscovery of the Aritmetica of Diophantus. In particular this is true of the manuals published in France, those of Scheubel, Peletier, Borrel and the three works by Gosselin. The definition and classification of equations established by Scheubel was not followed as such, but the idea of developing algebraic manuals around the explicit definition of equation, and around the classification of equations was influential.

It is in the same spirit that Peletier entitled the section from which we took the previous quotations De l'equacion, partie essancielle de l'Algebre.[325] On the other hand we

324 See Petri Rami, Arithmeticae libri duo, Basileae 1569, p.52: "Comparatio quantitatis in numeris est differentia vel ratio."

325 We read further in the same section: "Une Equacion se doet reduire a tele forme, que le nombre Cossique, s'il n'y an à qu'un, demeure seul d'une part, egal au reste de l'Equacion: E s'entand aussi, Quand il se trouvera une Equacion comprenant divers nombres Cossiques: que celui de plus

have already seen earlier some indications of the role attributed by Peletier to equation as a key notion of algebra. Further in the text, in the chapter about the setting of an equation, La grande Regle generale de l'Algebre, Peletier writes that everything that has come before, with all the various algebraic comparisions, is merely preparation for this part, and gives the following rule:

> Au lieu du Nombre inconnu que vous cherchez, metèz 1R : avec laquele faites votre discours selon la formalité de la Question proposée: tant qu'eyez trouuè vne Equacion convenable, e icelle reduite si besoin est. Puis, par le Nombre su sine maieur Cossique, diuiseèz la partie a lui egalee: ou en tireèz la Racine tele que montre le Sine. Et le Quociant qui prouiendra (si la division suffit) ou la Racine (si l'extraccion est necessere) sera le Nombre que vous cherchèz.

Before Stifel, the fact of putting a question into an equation had not been specified as an operation in itself. Stifel introduced this operation into the context of a research directed to a classification of equations. Later, this process was connected with the rediscovery of Diophantus. Gosselin, with his De arte Magna (Paris 1577), wrote the first algebraic manual which took into account the Arithmetica of Diophantus, both because Xylander had translated and published it two years earlier, and because Gosselin had a manuscript of the Greek text. Furthermore, into his algebraic theory, which included a new and better classification of equations (the first according to the degree of the unknown), Gosselin also integrated his philosophical vision of the role of algebra and of its structure. He writes:

> Finis huius scientiae est cognitio quantitatis ignotae, quam ut eliciamus, utimus aequatione tamquam medio.

The third book is entirely devoted to equations. Gosselin begins like Peletier with the

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grand denominacion, c'ét a dire, qui aura le plus grand sine, devra demeurer seul, egal au rest de l'Equacion: Ce qui se fera par transposicion, en cete sorte."(p.25). This passage could be compared with some of the prescriptions of the last Regulae.

statement Cum haec ars praestantissima in aequatione et laterum deductione tota fere consistat. But he goes on:

> opportunum visum est his omnibus expeditis ad aequationem tanquam ad apicem et fastigium huius scientiae devenire: haec enim est sine qua nihil conducant praecedentia, possit autem aliqua ratione sine illis consistere.

The definition that follows is particularly explicit:

> aequatio autem est duarum quantitatum diversi nominis et valoris ad unam aestimationem reductio, ut cum dicimus unum Quadratum aequari quatuor lateribus ...

We could multiply citations which develop the reflection begun by Peletier. We shall limit ourselves to mentioning the third work by Gosselin, De ratione discendae docendaeque mathematices, a praelectio or presentation of a university course, which has the style of an annotated table of contents of mathematical themes. A section of it is devoted to algebra, which is called subtilior arithmetica. He mentions again some of his theses:

> Finis scientiae, quantitates ignotae cognitio, media ad illum finem, aequatio vel aequalitas. (...) aequatio dicitur, cum aliquae quantitates diversi generis inter se aequale proferuntur.

After this, there follows a classification of equations, according to form or degree.

Ramus had published an Algebra anonymously (Paris, 1560); this Algebra probably had a considerable diffusion especially through the expanded edition of Lazar Schooner (Frankfurt, A. Wechel, 1592). Furthermore we know that the list suggested by Snell père to Beeckman probably included it, because for practical arithmetic Ramus and Clavius are mentioned.[326] The work is divided in numeratio (arithmetic of relative numbers) and aequatio.

The second book begins with the definition aequatio est qua figurati inter se

326 See AT, vol.X, p.29.

secundum hypothesis aequantur. For the classification, Ramus follows that of Scheubel. In conclusion we can observe that already before Viète's development of algebra, the algebraic tradition had elaborated two correlative notions, quaestio and equatio. A problem should be dealt with algebraically and an equation should be seen as an abstract form of problem. Algebra even made quaestio and aequatio the most important tools of its transformation from the abachos tradition to logistica, and also in its integration of the rediscovery of the classics and particularly of Diophantus.

As for Clavius, we find, even in the more accessible edition of 1612, many passages which illustrate how a discussion of the role of equations among the mathematical sciences was present also in teaching. In the proemio to Algebra (op. cit. vol. II) we find:

> Propositum sive scopus eius est, ut certam aliquam a sensuum cognitione, ac sensu secretam quantitatem exploraret, et tandem inter duos aliquos numeros aequalitate, sive aequatione comperta, deprehendat, atque demonstret.(p.3)

His formulation of the regula algebrae is the following:

> Pro numero incognito in quaestione ponatur radix una hoc modo, 1x. (Possunt etiam plures radices poni hoc modo 2x vel 3x etc. vel alius quidam numerus, pro commoditate quaestionis propositae.) Quae iuxte quaestionis tenorem examinetur, donec Aequatio aliqua inveniatur: Haec reducatur, si reductione opus fuerit: Deinde per numerum characteris Cossici maioris dividatur reliquus aequationis numerus. Nam vel Quotiens ipse erit numerus, qui quaerebatur, pretium scilicet radicis in principio positae, vel certe radix aliqua Quotientis numeri numerum, qui quaerebatur, notum reddet.(p.2O)

Clavius summarizes the passages indicated in the rule in a way that reminds us of the subdivision of parts in Viète's ars analytica:

> Habet autem regula haec quatuor partes. Prima est inventio Aequationis: Secunda Reductio Aequationis inventae: Tertia, Divisio alterius numeri Aequationis per numerum maioris characteris Cossici: Quarta et ultima, Extractio radicis alicuius ex Quotiente.(...) Est autem Aequatio, ut hic sumitur, nihil aliud, quam proportio aequalitatis inter duas quantitates, sive

res varie denominatas.(ibidem)

We can state, in conclusion, that Clavius is relevant in this context, both for the theoretical arrangement of the distinction between theorems and problems, and for the notion of equation. As for the use of the terms problema, quaestio, aenigma, no pattern is discernable. On the one hand he does not make use of the term quaestio in geometry, but in algebra he refers indifferently to quaestiones, problemata, aenigmata. We should recall also that he specifically discussed the possibility of applying algebraic methods not only to geometry but also to all the mathematical sciences, although this project was never completed. While the application of algebra to geometry was not an innovation with respect to his predecessors, its application to the various mathematical sciences was a new and important contribution.

### *4. **Brief overview of the critical literature***

This dimension of the text of the second book of the Regulae refers to two aspects of Descartes' cultural milieu, on the one hand, the growing importance of the theory of equations or rather the transformation of algebra from ars rei et census into a theory of equations,[327] and on the other hand the elaboration of the idea of scientific problem with respect to new ideas and practices in the sciences.

From the point of view of the history of science, the certainty that these processes took place cannot replace the investigation of how they actually took place, in particular, with respect to two traditions implied in the process, that is, algebra and Aristotelianism.

As for the first, the researches developed and promoted by Charles Schmitt have

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327 That this transformation took place is generally accepted. I take this formulation from M.S.Mahoney The beginnings of algebraic thought, in Gaukroger ed. op.cit. However, much remains to be done to give meaning to this general description.

already made clear how, in its later sixteenth-century form, university Aristotelianism was an important condition and context for formulation of the new sciences. As for algebra, recent studies, despite their neglect of the pre-Vietian tradition, can contribute to a better awareness of the lexicon. Recently Gaukroger,[328] illustrating the importance of the Regulae for the construction of the mathematical physics, has also stressed how first of all "Descartes' problem is to specify and realize the conditions under which physical problems can be posed mathematically." Gaukroger raises the issue we are dealing with, i.e. Descartes' elaboration of the notion of problem. As we see in the above sentence, however, the distinction between the present meaning of the term problem and the meaning which it may have had for Descartes tends to be obscured. I want to suggest that in order to see how he uses it, and how he sets the limiting conditions for its use in science, it is important to study the history of the term. If it is true that Descartes founded a mathematical physics by maintaining a program of reduction of all the mathematical disciplines and of natural philosophy to the simplicity and certainty of results typical of arithmetic and geometry, we may expect that he applied a typically mathematical term, such as problem, to all problems or questions of the sciences. As we have seen, the reality is a little more complex, and three terms, quaestio, difficultas, problema, are used as synonyms. However, the interpretation of the Regulae has a broad tradition in the history of philosophy. First of all we must recall that many critics have taken into consideration the relationship between the Regulae and the method (and the Discours de la Méthode), in particular the identification or the distinction between mathesis universalis and method. Recently J.L. Marion (Sur l'ontologie grise de Descartes, Paris, Vrin, 1975) has re-elaborated the thesis of G. Milhaud (Descartes savant, Paris, Alcan,

328 See S. Gaukroger, "Descartes' project for a mathematical physics", in S.Gaukroger, op.cit., p. 98, which has been very important to me in the development of this research.

1921) according to which mathesis universalis coincides with the Cartesian method. Even the relationship with the mathematical problem worked out in the Géométrie constitutes the kernel of the reflection on the Regulae (discussed by Beck, see above), while the Géométrie has been object, in the last years, of several studies.[329] Meanwhile, the text of the Regulae has been analyzed more rigorously, beginning with a new interpretation by J.P. Weber, (La constitution du text des "Regulae", Paris, Société d'édition d'enseignement supérieur, 1964), which concludes with the thesis that the text was composed in different layers. Then there are the important philological contributions made by the editions of Crapulli and Springmeyer, new translations into modern languages[330] and finally the already mentioned Lexique.[331] A recent revision of Weber's thesis is presented in the study by Schuster already mentioned. Particularly interesting for our perspective are the several works on Descartes'

329 See in particular M.Bos, "On the representation of curves in Descartes' Géométrie", in Archive for the history of exact sciences, 24, 1981, as well as his more extended study, extensively clarifying an important aspect of the Géométrie as well as a specific sense of the notion of "problema" in Descartes, "Arguments on motivation in the rise and decline of a mathematical theory: the 'construction of equations', 1637-ca.1750", Archive for history of exact sciences vol. 30, no. 3/4, 1984. Among the most recent works, see some of the studies of two conferences on the Discours: Le Discours et sa méthode, Actes du Colloque pour le 350e anniversaire du 'Discours de la méthode' (Paris, 28-30/I/1987), publiés sous la direction de N.Grimaldi e J.L.Marion, Paris 1987; also Atti del Convegno "Descartes: il Discorso sul metodo e i Saggi di questo metodo. 1637-1987", Roma 1989. Of great importance for the themes treated in this essay, as well as a serious attempt to connect the scientific understanding of Descartes' work and his metaphysics is the already mentioned collective work edited by Gaukroger. The most updated bibliography is in Giovanni Crapulli, Introduzione a Descartes, Bari 1988.

330 René Descartes, Règles utiles et claire pour la direction de l'esprit en la recherche de la vérité, traduction selon le lexique cartèsien, et annotations conceptuelles par J.L.Marion avec des notes mathématiques de P.Costabel, La Haye 1977; René Descartes, Regulae ad directionem ingenii, Kritisch revidiert und herausgegeben von H.Springmeyer und H.G.Zerkl, Hamburg 1972.

331 Besides the lexical studies, for which I refer to the already cited Crapulli 1988, pp.254-255. For this topic, I have also used E. Gilson, Index scolastico-cartésien, Paris 1913, 1979 r., which in any case does not mention any synonym of problem, as well as G. Crapulli, "Le note marginali latine nelle versioni olandesi di opere di Descartes di J.H. Glazemaker", in G. Crapulli - E. Giancotti Boscherini, Ricerche lessicali su opere di Descartes e Spinoza, Roma 1969.

mathematics by P. Costabel, and Gäbe's work on Descartes' earlier years.[332] Of specific importance for our discussion are the works by J.L. Marion and N. Bruyère. In the recent work by Bruyère.[333] he discusses various elements of the Ramusian heritage in Descartes, particularly in the text of the Regulae.

Marion's interpretation of the *Regulae* needs to be considered here in more detail. In his view, Descartes wrote the Regulae with Aristotle in mind as his interlocutor. As he indicates, the notion of the episteme underpinned by the Regulae overthrows the classification of the sciences according to the genos. I add that this involves the overthrowing of Aristotle's setting of the problems according to the genos, the pecularity and the accident. Descartes must therefore find a uniform way of formulating problems, one that would not make use of Aristotelian classification, and this way is suggested by the algebraic tradition: every problem, is in its analytically treatable form (that is algebraically) an equation. From quaestio, we come to the comparatio in the double sense of equation and of comparison of the two natures. This equation or comparison has the goal of knowing the one through the other. The secret of the art which allows the mind to know each thing starting from another is to be constrasted to the Aristotelian way of proceding, which refers to the *genos* of the being. Marion interprets Descartes' notion of question in terms of the idea of enumeration and this in an ontological way more than in a methodical or epistemical way. If, by contrast, we read numeration itself as preparation for the correct formulation of a problem or rather of a quaestio, and therefore we read numeration itself as preparation for

332 P. Costabel, Démarches originales du Descartes savant, Paris 1982; P. Costabel, "L'initiation mathématique de Descartes", Archives de philosophie, 4 1983; L. Gäbe, Descartes Selbstkritik. Untersuchungen der Philosophie des jungen Descartes, Hamburg 1972.

333 N. Bruyère, Méthode et dialectique dans l'oeuvre de La Ramée Paris, Vrin 1984, particularly at pages 385-396.

the establishment of an equation, the text should be put next to another passage of Aristotle, not mentioned by Marion. This is the already mentioned passage of the Posterior Analytics 98 a (see n.4). Descartes' answer to this new way of reducing problems to a standard form is contained in Regula XIII and more fully in Regula XVII. For in Regula XIII we have the reference to the simplification of quaestio and even specifically in quam minimas partes cum enumeratione dividenda. On the other hand, Regula XVII identifies the difficultas with reciprocal dependence of terms known and unknown. If what we have traced is actually the connection Descartes intended, we can say that he has profoundly transformed the notion of problem proposed by Aristotle. The problem is no longer defined by a relation between subject and predicate, but by an equality between operations.[334] Marion stresses the connection between enumeration and equation, also because as we know the only example of enumeration explicitly developed by Descartes is the one of succession of powers of one unknown (Crapulli 20,9; AT 384,21). Furthermore, he draws all the possible consequences from the elimination of the Aristotelian genos. However Marion does not, though according it some importance (Marion, 172), grant to quaestio the positive role that even Aristotle attributed to it (Posterior Analytics, 77 to 37, see n. 5). Not all the questions had that simple plausibility to which Marion refers (173). And for an interpretation of *quaestiones* that is not only plausible but scientific, we need only to think of the sixteenth-century debates, in which quaestio was the starting point for inventio. At the same time, inventio was a very important part of the scientific discourse, not so much of course in order to give knowledge

334 In other words, the logic of predicates by a propositional calculus; this calculus, in turn, is not conceived of as a logic, and in this sense Leibniz rightly maintained that Descartes did not draw the consequences from his own positions. The calculus of properties and of relations of equalities, a calculus without substances either Aristotelian or Leibnitian, but simply of equality and binary operations, is implied in the second book of the Regulae, and developed only in the context of the theory of equations in the *Géométrie*. What remains is the algebrization of the notion of problem.

a systematic character, but to guide natural intelligence from complex to simple truths. Marion, on the other hand, sees only the interrogative content of the erôtêma in Aristotle, and he maintains that the quaestio is not a truth or is not yet a truth and he goes as far as to interpret the unknown content, that is the unknown of the quaestio, as a Cartesian version of the "non certain" content of the erôtêma:

> "L'inconnu résiduel, mais irrémédiable, de l'erôtesis, de la demande, devient une inconnue provisoire et réductible."(Marion, p.173)

Without denying this meaning of erôtêma, we have already seen a few reasons for Descartes to have attributed a positive meaning to the term, and even the text suggests the new technical meaning. Descartes, grounding himself on Aristotelian comments and on the algebraic tradition, gave the term quaestio the positive meaning that Aristotle attributed to problema. This is indirectly confirmed by the fact that the interrogative meaning of question is completely abandoned in the text of the Regulae, while what is adopted is the meaning of problem, both in the Aristotelian and mathematical sense. In conclusion, Marion has widely argued that the system of genera reflected in the classification of sciences had necessarily to fall in connection with an ontology ordered only by the thinking substance. On the other hand, perhaps because of his desire to bring to light the transmutation of the Aristotelian system into the Cartesian, Marion glosses over the extent to which this passage was mediated by the sixteenth-century interpretations of Aristotle. So the notion of problema-quaestio, as other topoi of the sixteenth-century Aristotelianism, has not found its own place in Marion's argument, an argument which would have been strengthened by it.

## *5. CONCLUSION*

The notion of problem and its transformation is central in the text of the Regulae. For Descartes, who founded his view of the role of algebra on the consolidated tradition of the French manuals of Logistica, it was clear that all problems are in principle solvable, and therefore that the epistemological weight shifts towards the setting of problems. Since in the Regulae, Descartes dealt first of all with this aspect, we must not limit ourselves to considering the text as a program of algebrization oriented to the solution of problems. The point is that we solve all and only the problems which we know how to set. From the historical point of view, what interests us is not so much to see how Descartes proposed to treat problems mathematically, but how he was ready to conceive problems in algebraic terms, given the fact that a reduction of an equation was only the last phase of a process of adaptation from the idea of problem to the concrete question. Therefore we have tried to show in which senses the Cartesian model of problem was algebraic and how this conception accorded with a reform of Aristotelian logic.

For the first point, we started with the thesis (already formulated by Marion) according to which Descartes, who plans to constitute the mathematical sciences systematically, starting from the lumen naturae and therefore from the calculus (on signs) of simple ideas, found himself bound to replace the classificatory Aristotelian hierarchy and its logic. Descartes founds his version of logistic (which takes advantage of the introduction of unity and the notation for powers), on a reduction of all quantity in terms of measure and order. Measure refers to unity, and order to succession of powers of the unknown. Therefore it is possible to reduce the unknown quantity to operations on known quantities, according to the rules of algebra. The equation (comparatio) becomes therefore the center of the cognitive process, and equation is the quaestio itself, all the more because it guarantees both the elimination of whatever creates an obstacle, by the possibility of substitution, and at the same time, the preservation of what is important, insofar as the same order induces both the enumeration of problems according to complexity and the degrees of the equations which correspond to it. We have here a theory of scientific procedure, or rather a theory of the inventio founded on proceeding by problems, where the latter are no longer defined by Aristotelian ontology but by reciprocally relative terms without connections except those of enumeration in the order of complexity, that is the order of the comparatio, in the double sense of it. Furthermore, the study of sixteenth-century algebra and the algebra contemporary to Descartes seem to be able to explain his initial choice, that is to redefine the quaestio and to conceive it as an aequatio. In other words, the identification between quaestio and aequatio presented by Descartes as a point of departure is rather a point of arrival. The quaestio has the structure of an equation in the algebraic manuals, where it often reflects the widening of meaning with respect to the mathematical problem, because it can consist either of a problem or of a caso of merchant arithmetic, or again it could be considered as a specific application to any mathematical science of the rules found through general problems. To sixteenth-century algebrists Descartes owes, among other things, all the theory except the two innovations of unity and succession, at least until in the Géométrie, he extended remarkably the strength of the theory of equations.

Our conjecture on the interconnection between the three synonyms of problem in the Regulae has taken us on the one hand to the interconnection between the three synonyms in Aristotle, and on the other hand to the study of the algebraic meanings of the Cartesian terms. The fact that the connection between the Cartesian synonyms was by itself an object of reflection is shown by Toleto, in his commentary on the well-known beginning of the second book of the Posterior Analytics:

> "Adverte autem, quod graece non habetur vocabulum, quaestiones, sed zêtoúmena id est quaesita, (...) quaestiones vero dicuntur res eadem, secundum formam dubitationis, et interrogationis, voce, aut conceptu significatae, et ordinatae."[335]

We should add that Descartes himself used the term quaestio, or question, in the most common sense of that time, to mean a problem proposed by one mathematician to another, in letters written both prior to and contemporareously with the Regulae. In this text, on the contrary, he defines a specific technical meaning of quaestio, which is stable in the synonym difficultas, and which will also be transmitted to the Discours de la méthode as difficulté. This use distinguishes his approach from that of Fermat, the only contemporary mathematician who had reached and perhaps surpassed Descartes' specific competence and even his innovation. Fermat had already in 1628 written a version of his Isagoge and some results of his "méthode", which implies an extension of the theory of equations comparable to the one of Géométrie. What distinguishes Descartes from Fermat is the relationship with the reform of logic and the mathematical sciences. While Fermat adopted the program of the Logistica of Viète, who had planned to solve all problems in the mathematical sciences in a strict sense, Descartes widened the field of application of the new algebra both in the direction of logic, in a theory of method, and in the direction of an extention of the problems conceivable and also treatable mathematically, that is of all natural philosophy.

To Marion's conclusion, that Descartes with his Regulae went from the theme to the object, I would like to add that he went from the Aristotelian problêma to the algebraic quaestio, with inevitable consequences both for mathematics and for natural philosophy.

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335 Francisci Toleti Societatis Iesu Commentaria una cum quaestionibus in universam Aristotelis logicam, Venetiis, 16O7, p.211.

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